**St. Francis Xavier University**
**Department of Computer Science**

# CSCI 541: Theory of Computing
# Assignment 2
# Due November 29, 2022 at 8:15am

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**Assignment Regulations.**

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

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[8 marks] 1. Consider the language $L = \{\mathtt{d}^{2j}\mathtt{e}^{2\ell}\mathtt{f}^{j} \mid j \geq 1, \ell \geq 0\}$.

(a) Construct a pushdown automaton recognizing the language $L$. You should give both the elements of the tuple defining the pushdown automaton (i.e., $Q$, $\Sigma$, $\Gamma$, $\delta$, $q_0$, $F$) and an illustration of the pushdown automaton.

(b) Is your pushdown automaton from part (a) deterministic? If so, explain precisely what makes your pushdown automaton deterministic. If not, is it possible for the language $L$ to be recognized deterministically?

[6 marks] 2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. For each of the following languages, determine whether or not the language is context-free. If the language is context-free, then give a context-free grammar that generates the language. If the language is not context-free, then prove this using the pumping lemma.

(a) $L_1 = \{\mathtt{c}^{2i}\mathtt{b}^{k}\mathtt{a}^{3k}\mathtt{d}^{i} \mid i \geq 0, k \geq 1\}$.

(b) $L_2 = \{\mathtt{d}^{k}\mathtt{c}^{i}\mathtt{b}^{k}\mathtt{a}^{k} \mid i \geq 1, k \geq 1\}$

[6 marks] 3. Describe a deterministic single-tape Turing machine that decides the following language $L$ over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$:

$$L = \{w \in \Sigma^* \mid |w|_{\mathtt{a}} \leq |w|_{\mathtt{b}} \leq 2 \cdot |w|_{\mathtt{a}}\}.$$

Here, the notation $|w|_{\mathtt{a}}$ denotes the number of occurrences of the symbol $\mathtt{a}$ in the word $w$.

*Note.* You do not need to construct the Turing machine explicitly; you need only give a set of steps describing how some Turing machine $\mathcal{M}$ would decide the language $L$.

[4 marks] 4. Consider the following decision problem:

$SUB_{\mathsf{DFA}} = \{\langle \mathcal{B}, w\rangle \mid \mathcal{B}$ is a deterministic finite automaton and some word in $L(\mathcal{B})$ contains $w$ as a subword$\}$.

Prove that $SUB_{\mathsf{DFA}}$ is decidable by giving a decision algorithm.

*Hint.* You may find the fact that the class $\mathsf{DFA}$ is closed under the operation of intersection to be useful.

[6 marks] 5. Consider the following decision problem:

$$POW_{\mathsf{TM}} = \{\langle \mathcal{M}, w\rangle \mid \mathcal{M} \text{ is a Turing machine, } w \text{ is a word, and } \mathcal{M} \text{ accepts } w^k \text{ for all } k \geq 0\}.$$

Show that the problem $POW_{\mathsf{TM}}$ is undecidable.

*Hint.* Reduce from another undecidable problem for Turing machines.