St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Assignment 3
Due November 17, 2023 at 11:30am

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Consider the language

$$L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^{2j} \mid i \geq 1, j \geq 0\}.$$

Construct a pushdown automaton that recognizes the language L, then give a trace of the behaviour of your pushdown automaton on the input word `aabbcccc`. Your trace should list, at each step of the computation: the current state of the pushdown automaton, the current stack contents, and the remaining input word symbols.

[8 marks] 2. Consider the following grammar:

$$\begin{aligned} S &\to \mathtt{a}S\mathtt{a} \mid B \\ B &\to \mathtt{bb}C \mid \mathtt{bb} \\ C &\to \mathtt{c}C \mid \epsilon \end{aligned}$$

Convert this grammar to an equivalent grammar in Chomsky normal form.

[7 marks] 3. Consider the following context-free grammar G, where $V = \{R, S, T, X\}$, $\Sigma_G = \{\mathtt{a}, \mathtt{b}\}$, the start nonterminal is R, and the rule set contains the following rules:

$$\begin{aligned} R &\to XRX \mid S \\ S &\to \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a} \\ T &\to XTX \mid X \mid \epsilon \\ X &\to \mathtt{a} \mid \mathtt{b} \end{aligned}$$

Convert G to an equivalent pushdown automaton $\mathcal{M}$. You do not need to draw the pushdown automaton, you just need to give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

[7 marks] 4. Let $\Sigma = \{\mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}\}$. Using the pumping lemma for context-free languages, prove that the following language is not context-free:

$$L = \{\mathtt{b}^i\mathtt{c}^k\mathtt{d}^i \mid k \geq i \geq 0\} \cdot \{\mathtt{d}^m\mathtt{e}^{2m} \mid m \geq 0\}.$$

Note. The symbol $\cdot$ denotes concatenation; that is, words in L are of the form $\mathtt{b}^i\mathtt{c}^k\mathtt{d}^{i+m}\mathtt{e}^{2m}$.