**St. Francis Xavier University**
**Department of Computer Science**

# CSCI 356: Theory of Computing
# Assignment 4
# Due December 6, 2023 at 11:30am

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**Assignment Regulations.**

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

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[10 marks] 1. Construct a single-tape deterministic Turing machine $\mathcal{M}$ that performs the following "right shift" operation: given a word $w \in \{\mathtt{a}, \mathtt{b}\}^*$, $\mathcal{M}$ shifts the entire input word rightward by one cell on its input tape. After performing this shift, $\mathcal{M}$ halts and accepts.

For example, if $\mathcal{M}$ is given the input word `abba`, its tape at the beginning of the computation will look like

| ⋯ | a | b | b | a | ␣ | ␣ | ␣ | ⋯ |
|---|---|---|---|---|---|---|---|---|

and, after $\mathcal{M}$ halts and accepts, its tape will look like

| ⋯ | ␣ | a | b | b | a | ␣ | ␣ | ⋯ |
|---|---|---|---|---|---|---|---|---|

You should give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$. You may optionally draw the Turing machine to illustrate your construction.

[6 marks] 2. Consider the language $L_{\text{double}} = \{ww \mid w \in \{\mathtt{0}, \mathtt{1}\}^*\}$ of "doubled" binary words. This language is very similar to the language $L_{\text{double\#}}$, but without the delimiter symbol `#` separating the two halves of the input word.

Describe a (one-tape) deterministic Turing machine that recognizes the language $L_{\text{double}}$.

*Hint.* Use more than two symbols in your tape alphabet $\Gamma$.

*Note.* Unlike in the previous question, you do not need to construct the Turing machine explicitly. You need only give a set of steps describing how some Turing machine would recognize the language, similar to describing the pseudocode for an algorithm.

[6 marks] 3. Consider a variant of the Turing machine model where the transition function is defined as follows:

$$\delta : (Q \setminus \{q_{\text{accept}}, q_{\text{reject}}\}) \times \Gamma \to Q \times \Gamma \times \{R, N\}.$$

This variant machine is only able to move its input head rightward by one cell, or stay where it is (i.e., make "no move").

Does this variant machine recognize the same class of languages as an ordinary Turing machine? If so, explain why. If not, explain which class of languages it recognizes and justify your reasoning.

[8 marks] 4. (a) Consider the following decision problem:

$$EQ_{\mathsf{DFA,RE}} = \{\langle \mathcal{B}, \boldsymbol{r} \rangle \mid \mathcal{B} \text{ is a deterministic finite automaton, } \boldsymbol{r} \text{ is a regular expression, and } L(\mathcal{B}) = L(\boldsymbol{r})\}.$$

Prove that $EQ_{\mathsf{DFA,RE}}$ is decidable.

(b) A *useless state* in a Turing machine is a state that is never visited during the Turing machine's computation on any input word. Consider the following decision problem:

$$USELESS_{\mathsf{TM}} = \{\langle \mathcal{M}, q \rangle \mid q \text{ is a useless state in the Turing machine } \mathcal{M}\}.$$

Prove that $USELESS_{\mathsf{TM}}$ is undecidable by reducing from $E_{\mathsf{TM}}$ to $USELESS_{\mathsf{TM}}$.

[5 marks] 5. Choose your favourite topic from the course, and write a multiple-choice style question with one correct answer and 3–4 plausible-but-incorrect answers that tests a concept or notion related to that topic.

For inspiration, consider the multiple-choice style questions you saw on the midterm and practice midterm exams.