St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Final Examination
December 17, 2022
2:00pm–4:30pm

Student Name: ______________________________

Email Address: ______________________________

Instructor: T. J. Smith (Section 10)

Format:

The exam is 150 minutes long. The exam consists of 7 questions worth a total of 75 marks. The exam booklet contains 10 pages, including the cover page and one blank page at the back of the exam booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your exam booklet contains 10 pages. Do not detach any pages from your exam booklet.
4. Do not use any unauthorized reference materials or devices during this exam.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: ______________________________

Question	Marks	Score
1	20	
2	10	
3	5	
4	8	
5	10	
6	12	
7	10	
Total	75	

Multiple Choice

[20 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Which of the following models of computation recognizes exactly the class of regular languages?
- A. Acyclic finite automata.
- B. Nondeterministic finite automata with epsilon transitions.
- C. Deterministic pushdown automata.
- D. Single-tape Turing machines.

(b) Let $L = \{(\mathtt{a}^*\mathtt{b}^*\mathtt{c}^*)^k(\mathtt{a}^n\mathtt{b}^k\mathtt{c}^n) \mid n \neq k\}$. Which of the following words is in L?
- A. $\mathtt{abcabcabcabcabcabcabcabca}^2\mathtt{b}^8\mathtt{c}^3$
- B. $\mathtt{a}^3\mathtt{b}^2\mathtt{a}^3\mathtt{b}^2\mathtt{a}^3\mathtt{b}^2\mathtt{a}^3\mathtt{b}^2\mathtt{a}^7\mathtt{b}^3\mathtt{c}^7$
- C. $\mathtt{ab}^2\mathtt{c}^5\mathtt{ab}^2\mathtt{c}^5\mathtt{ab}^2\mathtt{c}^5\mathtt{a}^6\mathtt{b}^3\mathtt{c}^6$
- D. $\mathtt{a}^3\mathtt{bca}^3\mathtt{bca}^3\mathtt{bca}^2\mathtt{b}^2\mathtt{c}^2$

(c) Which of the following sets corresponds exactly to the class of context-free languages?
- A. All languages that have infinite size.
- B. All languages recognized by deterministic pushdown automata.
- C. All languages recognized by nondeterministic pushdown automata.
- D. All languages recognized by halting Turing machines.
- E. All languages recognized by Turing machines that may or may not halt.

(d) Let L and M be languages where $L \subseteq M$. Which of the following is **true**?
- A. If L is context-free, then M is context-free.
- B. If L is non-context-free, then M is non-context-free.
- C. If L is infinite, then M is non-context-free.
- D. If L is non-context-free, then M is infinite.

(e) Which of the following statements about Chomsky normal form grammars is **false**?
- A. The start nonterminal may appear on either side of any rule.
- B. For each nonterminal $A \neq S$, there exists no rule of the form $A \to \epsilon$.
- C. For nonterminals A and B, there exists no rule of the form $A \to B$.
- D. The grammar contains no rules with more than two symbols on the right-hand side.

(f) Which of the following statements is **false**?
   A. A Turing machine can write a blank space ␣ to its tape.
   B. The input alphabet Σ and the tape alphabet Γ of a Turing machine are always distinct.
   C. The input head of a Turing machine can remain on the same tape cell in two successive computation steps.
   D. A Turing machine can contain exactly one state.

(g) Which of the following statements is **true**?
   A. If a language L is semidecidable, then its complement $\overline{L}$ is also semidecidable.
   B. A language L is decidable if and only if both L and $\overline{L}$ are semidecidable.
   C. A language L is decidable if and only if L is not semidecidable.
   D. A language L is semidecidable if and only if L is undecidable.
   E. None of the above statements are true.

(h) Which of the following decision problems is decidable?
   A. $\{\langle \mathcal{M}, w\rangle \mid \mathcal{M}$ is a Turing machine that accepts input word $w\}$.
   B. $\{\langle G\rangle \mid G$ is a context-free grammar and $L(G) = \emptyset\}$.
   C. $\{\langle G\rangle \mid G$ is a context-free grammar and $L(G) = \Sigma^*\}$.
   D. $\{\langle \mathcal{M}, w\rangle \mid \mathcal{M}$ is a Turing machine that halts on input word $w\}$.
   E. None of the above decision problems are decidable.

(i) Suppose we have two decision problems A and B, and we know that $A \leq_m B$ and B is decidable. What can be said about A?
   A. A is always decidable.
   B. A may or may not be decidable depending on B.
   C. A is always undecidable.
   D. No such problems A and B can exist.
   E. Not enough information is given.

(j) Which of the following results is commonly known as the Cobham–Edmonds thesis?
   A. The classes P and NP are strictly separated.
   B. Exponential-time problems require more time to solve than polynomial-time problems.
   C. A problem can be feasibly computed if it can be computed in polynomial time.
   D. The class of polynomial-space problems is strictly larger than the class of polynomial-time problems.

Short Answer

[10 marks] 2. For each of the following questions, give a 1–2 sentence answer.

(a) What is Kleene's theorem? (Either state the theorem itself or describe what it tells us.)

(b) What is the difference between an ambiguous context-free grammar and an unambiguous context-free grammar?

(c) Suppose we have a Turing machine defined in the usual way, except its input head is allowed to make *three* types of moves: leftward, rightward, or stationary (where the input head does not move after reading some cell of the input tape). Is this "stationary Turing machine" equivalent in recognition power to the standard Turing machine? Why or why not?

(d) If $A \leq_m B$ and B is a regular language, does that imply that A is also a regular language? Why or why not?

(e) Let X be a complexity class. What does it mean for a decision problem to be X-hard? (Either describe hardness or give the condition for a decision problem to be hard.)

Long Answer

[5 marks] 3. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L = \{\mathtt{a}^{i+1}\mathtt{b}^{2k}\mathtt{c}^{2k}\mathtt{d}^{2k} \mid i \geq 0, k \geq 0\}.$$

This language is non-context-free, and we can use the pumping lemma to prove non-context-freeness. Below, the first and last parts of the proof are given to you. Fill in the rest of the proof.

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Assume by way of contradiction that L is context-free, and let p denote the pumping constant given by the pumping lemma. We choose the word $w =$ ______________________________. Clearly, $w \in L$ and $|w| \geq p$. Then, there exists a decomposition $w = uvxyz$ satisfying the three conditions of the pumping lemma.

We consider each case:

In all cases, one of the conditions of the pumping lemma is violated. As a consequence, the language cannot be context-free.

[8 marks] 4. Describe a deterministic single-tape Turing machine that decides the following language L over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$:

$$L = \{w \in \Sigma^* \mid |w|_{\mathtt{b}} = 2 \cdot |w|_{\mathtt{a}}\}.$$

Here, the notation $|w|_{\mathtt{a}}$ denotes the number of occurrences of the symbol $\mathtt{a}$ in the word w.

Note. You do not need to construct the Turing machine explicitly; you need only give a set of steps describing how some Turing machine $\mathcal{M}$ would decide the language L.

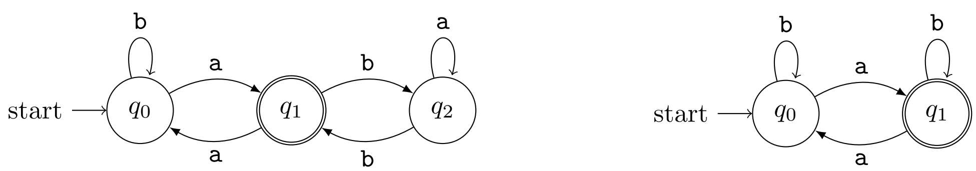
[10 marks] 5. Recall the following decision problems:

$$A_{\textsf{DFA}} = \{\langle \mathcal{B}, w\rangle \mid \mathcal{B} \text{ is a deterministic finite automaton that accepts input word } w\}$$
$$EQ_{\textsf{DFA}} = \{\langle \mathcal{B}, \mathcal{C}\rangle \mid \mathcal{B} \text{ and } \mathcal{C} \text{ are both deterministic finite automata and } L(\mathcal{B}) = L(\mathcal{C})\}$$

Furthermore, define a new "inclusion" decision problem as follows:

$$IN_{\textsf{DFA}} = \{\langle \mathcal{B}, \mathcal{C}\rangle \mid \mathcal{B} \text{ and } \mathcal{C} \text{ are both deterministic finite automata and } L(\mathcal{B}) \subseteq L(\mathcal{C})\}$$

Consider the following deterministic finite automata:



DFA $\mathcal{B}$ DFA $\mathcal{C}$

For each of the following questions, answer "yes" or "no" and justify your answer.

(a) Is $\langle \mathcal{B}, \texttt{abb}\rangle \in A_{\textsf{DFA}}$?

(b) Is $\langle \mathcal{C}, \texttt{bbaabbaab}\rangle \in A_{\textsf{DFA}}$?

(c) Is $\langle \mathcal{B}, \mathcal{C}\rangle \in EQ_{\textsf{DFA}}$?

(d) Is $\langle \mathcal{C}, \mathcal{B}\rangle \in IN_{\textsf{DFA}}$?

(e) Is $\langle \mathcal{C}, \texttt{ab}^n\rangle \in IN_{\textsf{DFA}}$?

[12 marks] 6. (a) Consider the following decision problem:

$$SUB_{\mathsf{TM/DFA}} = \{\langle \mathcal{M} \rangle \mid \mathcal{M} \text{ is a Turing machine and there exists a DFA } \mathcal{A} \text{ such that } L(\mathcal{M}) \subseteq L(\mathcal{A})\}.$$

Is $SUB_{\mathsf{TM/DFA}}$ decidable or undecidable? If the decision problem is decidable, give a decision algorithm that decides the problem. If the decision problem is undecidable, explain why.

(b) Let L be a semidecidable language, and assume that there exists a reduction $L \leq_m \overline{L}$ (where $\overline{L}$ denotes the complement of L).

Prove that L is decidable.

[10 marks] 7. Consider the following decision problem:

BOUNDED-LENGTH-PATH
Given: an undirected graph $G = (V, E)$, two vertices $a, b \in V$, and an integer $k \geq 1$
Determine: whether a path of length at most k exists from vertex a to vertex b

Show that BOUNDED-LENGTH-PATH is in P.

[2 marks] *Bonus.* What was your favourite part of this course, and why?

This blank page may be used for rough work.