St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Midterm Examination
October 21, 2022
11:15am–12:05pm

Student Name: ______________________________

Email Address: ______________________________

Instructor: T. J. Smith (Section 10)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: ______________________________

Question	Marks	Score
1	5	
2	6	
3	6	
4	8	
Total	25	

Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Let $A = \{\epsilon, \mathtt{ab}, \mathtt{ba}\}$ and $B = \{\epsilon, \mathtt{a}, \mathtt{ba}\}$. The concatenation of these languages, AB, corresponds to which of the following sets?

A. $\{\mathtt{a}, \mathtt{b}, \mathtt{ab}, \mathtt{ba}, \mathtt{aba}, \mathtt{ba}, \mathtt{ba}\}$
B. $\{\epsilon, \mathtt{a}, \mathtt{ab}, \mathtt{ba}, \mathtt{aba}, \mathtt{aab}, \mathtt{abab}, \mathtt{baab}\}$
C. $\{\epsilon, \mathtt{a}, \mathtt{ab}, \mathtt{ba}, \mathtt{aba}, \mathtt{baa}, \mathtt{abba}, \mathtt{baba}\}$
D. $\{\epsilon, \mathtt{b}, \mathtt{ab}, \mathtt{ba}, \mathtt{bab}, \mathtt{bab}, \mathtt{baa}, \mathtt{abba}, \mathtt{baba}\}$

(b) Let L be the language corresponding to the regular expression $(\mathtt{a} + \mathtt{aab} + \mathtt{aaab})^* \cdot (\mathtt{aa} + \mathtt{aaa})$. Which of the following words is in L?

A. $\mathtt{aaaabaa} \in L$
B. $\mathtt{aaabbaa} \in L$
C. $\mathtt{aaabaaba} \in L$
D. $\mathtt{aababaabaa} \in L$
E. None of the above.

(c) Let A and B be regular languages over the same alphabet Σ. Which of the following statements is true?

A. $A \cup B$ is always regular.
B. $A \cup B$ is never regular.
C. $A \cup B$ is sometimes regular, but not always.
D. $A \cup B$ cannot exist.

(d) Which of the following statements is true?

A. Every regular language is finite.
B. Every regular language is infinite.
C. Some regular languages are finite and some regular languages are infinite.
D. Finite automata can recognize only finite regular languages.

(e) Let $L = \{\mathtt{a}^i\mathtt{b}^{2k} \mid 0 \leq i \leq k\}$. Which of the following grammars generates the language L?

A. $S \to \mathtt{abb}S \mid \mathtt{bb}S \mid \epsilon$
B. $S \to \mathtt{a}S\mathtt{bb} \mid \epsilon$
C. $S \to \mathtt{a}S\mathtt{bb} \mid \mathtt{a}S\mathtt{bbb} \mid \epsilon$
D. $S \to \mathtt{a}S\mathtt{bb} \mid S\mathtt{bb} \mid \epsilon$
E. None of the above.

Short Answer

[6 marks] 2. (a) Given each of the following language descriptions, show how to define the language using the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$, ϵ, and the operations of union ($\cup$), concatenation ($\cdot$), and Kleene star (*). Note that you do not necessarily need to use all of these elements in each of your answers.

i. All words containing an odd number of occurrences of `0`.

ii. All words that have even length **and** contain `010` as a subword.

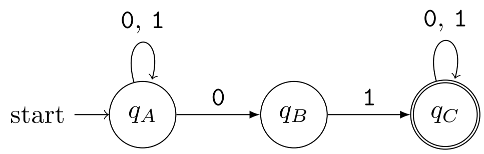
(b) Let $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$. Consider the language

$$L = \{\texttt{a}^{3i}\texttt{b}^{4i}\texttt{c}^{3k}\texttt{d}^{2k} \mid i \geq 1, k \geq 1\}.$$

i. Give a context-free grammar that generates words belonging to the language L.

ii. Using your grammar from part (b)i, give a derivation of the word $w = \texttt{a}^3\texttt{b}^4\texttt{c}^9\texttt{d}^6$.

[6 marks] 3. Consider the following nondeterministic finite automaton:



Using the subset construction, give the transition table for this nondeterministic finite automaton and use the information from your transition table to draw the equivalent **deterministic** finite automaton.

[8 marks] 4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L = \{\mathtt{a}^i \mathtt{b}^k \mathtt{c}^m \mathtt{d}^r \mid i \geq m \geq 0, k \geq 0, r \geq 0\}.$$

This language is nonregular, and we can use the pumping lemma to prove nonregularity. Below, the first and last parts of the proof are given to you. Fill in the rest of the proof.

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Assume by way of contradiction that L is regular, and let p denote the pumping constant given by the pumping lemma. We choose the word $w =$ ______________________________. Clearly, $w \in L$ and $|w| \geq p$. Then, there exists a decomposition $w = xyz$ satisfying the three conditions of the pumping lemma.

Thus, not all conditions of the pumping lemma have been satisfied, and we have reached a contradiction. As a consequence, the language L cannot be regular.

This blank page may be used for rough work.