St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 2
Due November 2, 2023 at 1:30pm

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.

- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.

- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Prove that if $\mathsf{NSPACE}(n) \subseteq \mathsf{NP}$, then $\mathsf{NP} = \mathsf{PSPACE}$.

Hints. (i) What do we know about the relationship between NP and PSPACE? (ii) Use Savitch's theorem.

[6 marks] 2. (a) Prove that every PSPACE-hard decision problem is also NP-hard.

(b) Suppose that every NP-hard decision problem were also PSPACE-hard. What would this imply? Explain your reasoning.

[8 marks] 3. Prove that the complexity class coNP is closed under polynomial-time reductions. That is, show that if $A \leq_m^P B$ and $B \in \mathsf{coNP}$, then $A \in \mathsf{coNP}$.

[10 marks] 4. Suppose you have a polynomial-time Las Vegas randomized algorithm solving a given problem. Show that you can always convert this to a polynomial-time Monte Carlo randomized algorithm with one-sided error solving the same problem.

Hint. To prove the probability that your Monte Carlo algorithm produces a correct answer, you may find *Markov's inequality* useful: given a nonnegative random variable X and a value $a > 0$, we have that $\mathbb{P}[X \geq a] \leq \mathbb{E}[X]/a$.

[8 marks] 5. Explain why the complexity class RP is not known to be closed under complement.

More specifically, suppose we have a probabilistic Turing machine $\mathcal{M}$ that recognizes a language $L \in \mathsf{RP}$. Explain what goes wrong if we try to modify $\mathcal{M}$ to recognize the language $\overline{L}$ instead.