St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 4
Due December 5, 2024 at 11:30am

Assignment Regulations.

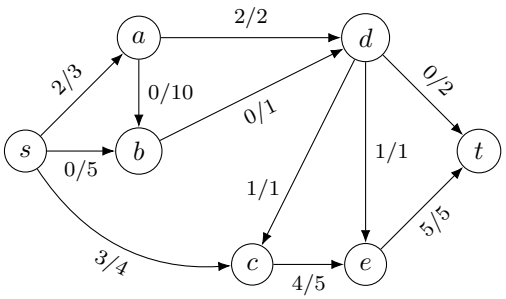
- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[7 marks] 1. A *common subsequence* of three strings X, Y, and Z is a string that is a subsequence of each of X, Y, and Z. For example, if $X =$ `postfaxed`, $Y =$ `starfoxes`, and $Z =$ `bestflix`, then one of the common subsequences shared by these strings is `stfx`.

Suppose that X, Y, and Z each have length n. Consider the problem of finding the length of the longest common subsequence of $X[1..n]$, $Y[1..n]$, and $Z[1..n]$, and give the Bellman equation that models this problem. Describe each case of your Bellman equation.

Hint. The problem of sequence alignment for two strings is similar to the problem of finding the longest common subsequence of three strings. Use the Bellman equation we established for sequence alignment as a guide.

[10 marks] 2. Consider the following flow network.



(a) What is the value of the flow shown in the network? Is this a max flow? If it is, explain why. If it is not, find a max flow.

(b) Find a min cut in the flow network, and give the capacity of your min cut.

[8 marks] 3. The university is in the midst of updating the campus wireless network, and to make sure everything works as expected, IT Services decides to test the new network with a group of n students. Each student, represented by an (x, y) coordinate specifying their location on campus, will connect to one of several base stations, each also represented by an (x, y) coordinate specifying its location on campus.

Each student connects to exactly one base station. This connection is constrained by two factors: a range parameter r, meaning that the student must be within distance r of the base station, and a load parameter ℓ, meaning that the base station cannot handle more than ℓ students.

Given the coordinates of the n students, the coordinates of the base stations, the range parameters of each student, and the load parameters of each base station, describe an algorithm to decide whether all n students can be simultaneously connected to some base station (not necessarily the same one), subject to the given range and load conditions.

You may describe your algorithm in either plain English or pseudocode. Your algorithm must run in polynomial time, but you do not need to provide a justification for your algorithm's runtime.

Hint. Model this scenario as a network flow problem.

[5 marks] 4. Choose your favourite topic from the course, and write a multiple-choice style question with one correct answer and 3–4 plausible-but-incorrect answers that tests a concept or notion related to that topic.

For inspiration, consider the multiple-choice style questions you saw on the midterm and practice midterm exams.