

**St. Francis Xavier University**  
**Department of Computer Science**  
**CSCI 356: Theory of Computing**  
**Assignment 1**  
**Due September 25, 2024 at 12:30pm**

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**Assignment Regulations.**

- This assignment must be completed individually.
  - Please include your full name and email address on your submission.
  - You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.
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- [8 marks] 1. (a) For each of the following languages over the alphabet  $\Sigma = \{0, 1\}$ , give a regular expression matching exactly that language.
- i.  $L_1 = \{w \mid w \text{ does not contain three consecutive 0s}\}$ .
  - ii.  $L_2 = \{w \mid \text{no } 0 \text{ appears after three consecutive 1s}\}$ .
- (b) For each of the following regular expressions, give a 1–2 sentence description of the language matched by that regular expression.
- i.  $r_1 = (0^*10^*10^*)^*$ .
  - ii.  $r_2 = (0 + 1)^* \emptyset 1$ .

- [5 marks] 2. Let  $\Sigma = \left\{ \begin{smallmatrix} 0 & 0 \\ 1 & 1 \end{smallmatrix}, \begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix} \right\}$ . Suppose that, when we concatenate these symbols, the top row and the bottom row of the concatenation each correspond to a binary number. For example, in the word  $w = \begin{smallmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{smallmatrix}$ , the top binary number is  $101 = 5$  and the bottom binary number is  $011 = 3$ .

Consider the language

$$L_{t>b} = \{w \in \Sigma^+ \mid \text{the top row of } w \text{ is a larger binary number than the bottom row of } w\}.$$

For example, the word  $w = \begin{smallmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{smallmatrix}$  is in the language  $L_{t>b}$ , since  $5 > 3$ , while the word  $w' = \begin{smallmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{smallmatrix}$  is not in the language  $L_{t>b}$ , since  $10 < 15$ .

Prove that  $L_{t>b}$  is a regular language by writing a regular expression that matches exactly this language.

*Note.* Each symbol must be treated as a unit. When you are reading (for example) the symbol  $\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}$ , you are only reading a single symbol; you cannot independently read either the top or the bottom bit.

- [6 marks] 3. Let  $\Sigma = \{0, 1\}$ . For each of the following languages, give a *deterministic* finite automaton recognizing exactly that language.
- (a)  $L_1 = \{w \mid w \neq 111\}$ .
  - (b)  $L_2 = \{w \mid \text{all occurrences of } 1 \text{ in } w \text{ appear before an occurrence of } 000\}$ .
  - (c)  $L_3 = \{w \mid w \in \Sigma^+\}$ .

- [6 marks] 4. Consider the following language over the alphabet  $\Sigma = \{0, 1\}$ :

$$L = \{w \mid w \text{ both starts and ends with } 0\}.$$

- (a) Construct a nondeterministic finite automaton recognizing the language  $L$ .
- (b) Convert your nondeterministic finite automaton from part (a) to a deterministic finite automaton using the subset construction. Show all your work.
- (c) Convert either your NFA from part (a) or your DFA from part (b) to an equivalent regular expression using the state elimination algorithm. Show all your work.