St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Assignment 2
Due October 9, 2024 at 12:30pm

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[7 marks] 1. Consider the following regular expression over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$:

$$(\mathtt{1} + \mathtt{0})^*(\mathtt{11} + \mathtt{00}).$$

Using the McNaughton–Yamada–Thompson algorithm, convert this regular expression to a finite automaton recognizing the same regular language. Show all your work in addition to giving the finite automaton. You do not need to remove epsilon transitions or determinize the finite automaton.

[6 marks] 2. Let $w = a_1 a_2 \dots a_{2n}$ be an even-length word where $a_i \in \Sigma$ for all i. Consider the following operation:

$$\text{pull}(w) = a_1 a_3 a_5 \dots a_{2n-1} a_2 a_4 \dots a_{2n}.$$

We can extend this operation from words to languages by taking $\text{pull}(L) = \{\text{pull}(w) \mid w \in L\}$.

Is it always the case that if L is a regular language, then $\text{pull}(L)$ is a regular language? If so, give a proof of this statement. If not, give a counterexample where L is regular but $\text{pull}(L)$ is not.

[6 marks] 3. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Using the pumping lemma for regular languages, prove that the following language is not regular:

$$L = \{\mathtt{a}^i \mathtt{b}^k \mathtt{c}^m \mathtt{d}^r \mid i \geq m \geq 0, k \geq 0, r \geq 0\}.$$

[6 marks] 4. Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. *Without* using the pumping lemma, prove that the following language is not regular:

$$L = \{w \mid w \neq xx \text{ for any } x \in \Sigma^*\}.$$