

St. Francis Xavier University
Department of Computer Science
CSCI 356: Theory of Computing
Assignment 3
Due November 15, 2024 at 11:30am

Assignment Regulations.

- This assignment must be completed individually.
 - Please include your full name and email address on your submission.
 - You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.
-

- [8 marks] 1. (a) Consider the context-free language

$$L_1 = \{x^i y^j z^k \mid i + k = j\}.$$

Construct a pushdown automaton that recognizes this language.

Hint. How can we match the sum of the counts of xs and zs to the count of ys using the stack of the pushdown automaton? Use nondeterminism.

- (b) Consider the context-free language

$$L_2 = \{a^{2i+1} b^{k+1} c^{2k+1} d^{m+1} \mid i \geq 1, k \geq 1, m \geq 1\}.$$

Give a context-free grammar that generates this language.

- (c) Consider the context-free grammar

$$\begin{aligned} S &\rightarrow 0A10 \mid B10 \\ A &\rightarrow 00 \mid \epsilon \\ B &\rightarrow A0 \mid B1 \end{aligned}$$

Show that this context-free grammar is ambiguous by finding a word in the language of the grammar that has more than one leftmost derivation.

- [6 marks] 2. Consider the following context-free grammar G , where $V = \{S, A\}$, $\Sigma_G = \{0, 1\}$, the start nonterminal is S , and the rule set contains the following rules:

$$\begin{aligned} S &\rightarrow AS \mid \epsilon \\ A &\rightarrow 0A1 \mid A1 \mid 01 \end{aligned}$$

Convert G to an equivalent pushdown automaton \mathcal{N} . You do not need to draw the pushdown automaton, though you can if you like; you just need to give each component of the tuple $\mathcal{N} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

- [5 marks] 3. Let $\Sigma = \{a, b, c\}$. Using the pumping lemma for context-free languages, prove that the following language is not context-free:

$$L = \{a^k b^{2k} c^i \mid k \geq i \geq 1\}.$$

- [6 marks] 4. Consider the Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_{\text{accept}}, q_{\text{reject}})$, where $Q = \{q_0, q_1, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{\mathbf{a}, \mathbf{b}\}$, $\Gamma = \{\mathbf{a}, \mathbf{b}, \sqcup\}$, and δ consists of the following transitions:

$$\delta(q_0, \mathbf{a}) = (q_0, \mathbf{b}, R)$$

$$\delta(q_0, \mathbf{b}) = (q_1, \mathbf{b}, L)$$

$$\delta(q_1, \mathbf{b}) = (q_0, \mathbf{b}, R)$$

$$\delta(q_0, \sqcup) = (q_{\text{accept}}, \sqcup, L)$$

- (a) Give the sequence of configurations of \mathcal{M} when it is given the word **aaaa** as input. Does \mathcal{M} accept this word?
- (b) Give the sequence of configurations of \mathcal{M} when it is given the word **abba** as input. Does \mathcal{M} accept this word?
- (c) What language does \mathcal{M} recognize? Give a brief justification for your answer.