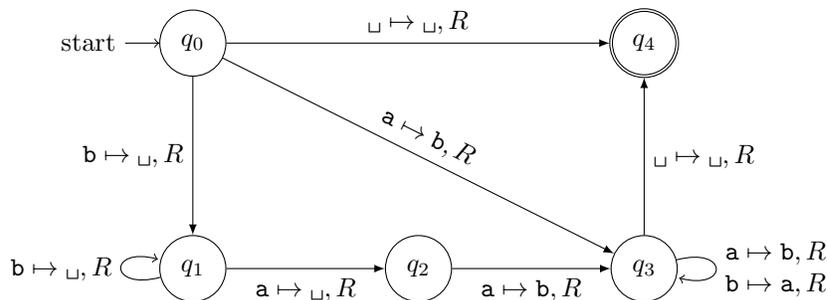


**St. Francis Xavier University**  
**Department of Computer Science**  
**CSCI 356: Theory of Computing**  
**Assignment 4**  
**Due November 29, 2024 at 11:30am**

**Assignment Regulations.**

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[4 marks] 1. Let  $\Sigma = \{a, b\}$ , and consider the following Turing machine  $\mathcal{M}$ :



- (a) Give an example of an input word  $w$  that  $\mathcal{M}$  would accept, and give the sequence of configurations of  $\mathcal{M}$  on  $w$ .
- (b) What does  $\mathcal{M}$  do to its input word  $w$ ? (You need not describe this as a language; a plain English description will suffice.)

[8 marks] 2. For each of the following problems, design a single-tape deterministic Turing machine that solves the problem. In your answers, you should give each component of the tuple  $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ . You may optionally draw the Turing machines to illustrate your constructions.

- (a) Given an input word  $w$  over the alphabet  $\Sigma = \{0, 1\}$  that represents a binary number, decide whether  $w$  is odd.
- (b) Given an input word  $w$  over the alphabet  $\Sigma = \{0, 1\}$  that represents a binary number, write the binary number representing the value  $2 \times w$  to the tape and then halt.

[6 marks] 3. By our definition of a Turing machine's transition function, the input head can make one leftward move or one rightward move on each computation step. We could optionally specify a "no move" transition  $N$ , where the input head neither moves left nor right (i.e., it stays on the same cell of the input tape). However, such a modification would not affect the recognition power of the Turing machine.

Explain briefly how we could simulate a "no move" transition of the form  $\delta(q_i, c) = (q_j, b, N)$  using only leftward and rightward moves.

- [7 marks] 4. Let  $\Sigma = \{0, 1\}$ . Consider the following decision problem:

$ZEROS_{RE} = \{\langle r \rangle \mid r \text{ is a regular expression over } \Sigma \text{ and all words in } L(r) \text{ contain } 0000 \text{ as a subword}\}$ .

Prove that  $ZEROS_{RE}$  is decidable by giving a decision algorithm.

*Hint.* Since regular expressions and finite automata are equivalent, use what we know about closure properties and decidability properties for the class DFA.

- [5 marks] 5. Choose your favourite topic from the course, and write a multiple-choice style question with one correct answer and 3–4 plausible-but-incorrect answers that tests a concept or notion related to that topic.

For inspiration, consider the multiple-choice style questions you saw on the midterm and practice midterm exams.