St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Midterm Examination
October 18, 2023
12:30pm–1:20pm

Student Name: _____

Email Address: _____

Instructor: T. J. Smith (Section 10)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: _____

Question	Marks	Score
1	5	
2	7	
3	7	
4	6	
Total	25	

Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Let L be the language corresponding to the regular expression $(\mathtt{a}^*\mathtt{b}+\mathtt{b}\mathtt{c}^*\mathtt{a})^*$. Which of the following words is **not** in L?

A. $\mathtt{bcababa}$.
B. $\mathtt{abbcaca}$.
C. $\mathtt{abaabbbca}$.
D. $\mathtt{babab}$.
E. All of the above words are in L.

(b) Which of the following statements is true for all languages L?

A. $L \cdot L = L$.
B. $L \cdot L \neq L$.
C. $L \cdot \{\epsilon\} = L$.
D. $L \cdot \emptyset = L$.
E. None of the above.

(c) Which of the following statements is true?

A. Every infinite language is regular.
B. Every infinite language is nonregular.
C. Every nonregular language is finite.
D. Every nonregular language is infinite.
E. None of the above.

(d) Let $V = \{S\}$ and $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. What language is generated by the following grammar?

$$S \to \mathtt{aa}S\mathtt{b} \mid \mathtt{a}S\mathtt{b} \mid \epsilon$$

A. $\{\mathtt{a}^i\mathtt{b}^k \mid 0 \leq k \leq i \leq 2k\}$.
B. $\{\mathtt{a}^i\mathtt{b}^k \mid 0 \leq i \leq 2k\}$.
C. $\{\mathtt{a}^{2i}\mathtt{b}^k \mid 0 \leq i \leq k\}$.
D. $\{\mathtt{a}^{2i}\mathtt{b}^k \mid i \geq 0, k \geq 0\}$.

(e) Which of the following is **not** a step in the process to convert a context-free grammar to Chomsky normal form?

A. Remove all rules of the form $A \to B$, where $A, B \in V$.
B. Remove all rules where some nonterminal symbol appears before a terminal symbol on the right-hand side.
C. Remove all rules with more than two nonterminal or terminal symbols on the right-hand side.
D. Remove all rules of the form $A \to \epsilon$, where $A \neq S$.

Short Answer

[7 marks] 2. (a) Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. Consider the languages $A = \{\mathtt{1}, \mathtt{10}, \mathtt{010}\}$ and $B = \{\mathtt{1}, \mathtt{01}\}$. List explicitly each element of the following languages.

i. $A \cdot B$

ii. $B \cdot A$

iii. $B \cup A$

(b) Let $V = \{S\}$ and $\Sigma = \{\mathtt{x}, \mathtt{z}\}$, and consider the grammar

$$S \to \mathtt{xx}S\mathtt{z} \mid \mathtt{x}S\mathtt{z} \mid \mathtt{z}.$$

i. Give a parse tree for the word xxxzzz.

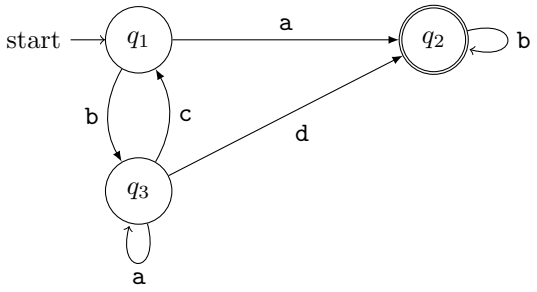
ii. Is this grammar ambiguous or unambiguous? Justify your answer.

iii. Is the language generated by this grammar regular or context-free? Select exactly one answer by circling the associated letter. No justification is necessary.

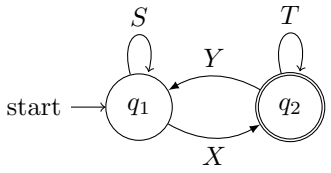
A. Regular

B. Context-free

[7 marks] 3. Consider the following deterministic finite automaton:



Using the state elimination algorithm to convert a deterministic finite automaton to a regular expression, we remove state q_3 to obtain the following finite automaton with transitions labelled by regular expressions:



(a) What regular expressions correspond to each of the labels S, T, X, and Y in the above finite automaton?

(b) What is the overall regular expression corresponding to the above finite automaton?

[6 marks] 4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L = \{\mathtt{a}^{2i}\mathtt{b}^{j}\mathtt{c}^{k}\mathtt{d}^{j+1} \mid i \geq 0, j \geq 0, k \geq 0\}.$$

This language is nonregular, and we can use the pumping lemma to prove nonregularity. Below, the first and last parts of the proof are given to you. Fill in the rest of the proof.

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Assume by way of contradiction that L is regular, and let p denote the pumping constant given by the pumping lemma. We choose the word $w =$ _____. Clearly, $w \in L$ and $|w| \geq p$. Then, there exists a decomposition $w = xyz$ satisfying the three conditions of the pumping lemma.

Thus, not all conditions of the pumping lemma have been satisfied, and we have reached a contradiction. As a consequence, the language L cannot be regular.

This blank page may be used for rough work.