St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 1
Due September 18, 2025 at 11:30am

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[7 marks] 1. Like many good humans, Prof. Smith is a cat person, and so are the people in his life. Each of these people has a preference for their favourite cats, and since cats are highly intelligent creatures, each cat has a preference for their favourite people. Suppose we have the following two preference lists, where the list on the left is each person's preference for cats ordered from most to least favourite, and the list on the right is each cat's preference for people ordered from most to least favourite:

Taylor	Treelo	Jesse	Snuffles	Darla
Madhavi	Darla	Jesse	Treelo	Snuffles
Sean	Darla	Snuffles	Treelo	Jesse
Brooke	Jesse	Treelo	Snuffles	Darla

Treelo	Taylor	Madhavi	Sean	Brooke
Jesse	Brooke	Madhavi	Taylor	Sean
Darla	Brooke	Sean	Taylor	Madhavi
Snuffles	Sean	Taylor	Madhavi	Brooke

Using the Gale–Shapley algorithm, find a stable matching that is person-optimal. Show all your work.

(If you like, you may implement the Gale–Shapley algorithm in the programming language of your choice, but your answer must include a copy of your source code and a verbose listing of each step of your implementation's output.)

[6 marks] 2. In our discussion of the stable matching problem, we assumed that our preference lists were totally ordered: in a given list, the first option was preferred over the second option, the second over the third, and so on. Here, we will consider a variant of the problem where we allow for indifference between options.

Suppose we have a set S of n students and a set H of n hospitals, with their associated preference lists, but now either list can indicate a tie between two students or two hospitals. As an example, when $n = 4$, one hospital could rank student s_1 first, rank both students s_2 and s_3 second (meaning that the hospital has no preference between these two students), and rank student s_4 last. As before, we say that hospital h *prefers* student s over student s' if s is ranked strictly higher than s' on the preference list of h (i.e., s and s' are not tied).

Say that a perfect matching M has *strong instability* if there exists a student s and a hospital h such that both s and h prefer the other to their assigned match in M.

Does there always exist a perfect matching with no strong instability? If so, give a procedure that is guaranteed to find a perfect matching with no strong instability. If not, give an example of a set of students S and a set of hospitals H whose preference lists are such that every perfect matching has a strong instability.

[6 marks] 3. For each of the following blocks of pseudocode, give as tight a bound as possible on the time complexity of the pseudocode using Big-O notation. You do not need to give a formal proof, but you should give justifications for each of your answers. You may assume that n is a positive integer.

(a)
```
x = 0
for i from 1 to n + 25:
    x = x * 19
    for j from 355 to 2025:
        for k from 4i to 5i:
            x = x * 93
```

(b)
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for (i = 1, i < n, i *= 2):
    for (j = n, j > 0, j /= 2):
        for (k = j, k < n, k += 2):
            s = s + (i + j * k)
```

[6 marks] 4. Arrange the following functions in order from slowest growth rate to fastest growth rate. If function $f(n)$ is in position i of your list and function $g(n)$ is in position $i+1$ of your list, then this is equivalent to saying that $f(n) \in O(g(n))$. Give a brief justification for your ordering.

$$\begin{array}{ll} f_1 = 4^{\log(n)} & f_4 = (3/2)^{n+1} \\ f_2 = \mathrm{e}^n & f_5 = n + n\log(n) \\ f_3 = \ln(n) & f_6 = n \cdot 2^n \end{array}$$