

**St. Francis Xavier University
Department of Computer Science**

**CSCI 355: Algorithm Design and Analysis
Final Examination
December 11, 2024
2:00pm–4:30pm**

Student Name: _____

Email Address: _____

Instructor: T. J. Smith (Section 10)

Format:

The exam is 150 minutes long. The exam consists of 6 questions worth a total of 70 marks. The exam booklet contains 9 pages, including the cover page and one blank page at the back of the exam booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your exam booklet contains 9 pages. Do not detach any pages from your exam booklet.
4. Do not use any unauthorized reference materials or devices during this exam.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Question	Marks	Score
1	10	
2	10	
3	14	
4	10	
5	12	
6	14	
Total	70	

Signature: _____

Multiple Choice

[10 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

- (a) Four students are feeling inspired after receiving their X-Rings, and are planning to go to graduate school to study algorithms. The following preference lists indicate the students' future preferences, as well as the preferences of each school:

	1st	2nd	3rd	4th		1st	2nd	3rd	4th
Tarry	StFX	UofT	UVic	Columbia	StFX	Tarry	CJ	Godson	Hevan
Hevan	UofT	Columbia	UVic	StFX	UofT	Godson	Hevan	Tarry	CJ
Godson	Columbia	UofT	UVic	StFX	Columbia	Tarry	Hevan	CJ	Godson
CJ	UVic	Columbia	StFX	UofT	UVic	CJ	Godson	Tarry	Hevan

Which of the following is **not** a stable pair according to these preference lists? (Assume the pairings are school-optimal.)

- A. CJ–UVic
 - B. Hevan–Columbia
 - C. Godson–UofT
 - D. Tarry–Columbia
- (b) Consider an instance of the scheduling problem solved using the earliest-deadline-first heuristic. Which of the following properties is guaranteed to be satisfied by the optimal schedule?
- A. The optimal schedule completes all jobs on time.
 - B. The optimal schedule contains one inversion.
 - C. The optimal schedule has no idle time.
 - D. The optimal schedule's total lateness is no worse than the maximum lateness squared.
- (c) Suppose you have a set of errands to run at a set of stores. Living in small-town Nova Scotia, these stores tend to have strange operating hours. Given each of the stores' opening times and closing times, which heuristic will produce the largest subset of errands you can complete?
- A. Earliest open time first
 - B. Earliest close time first
 - C. Shortest total opening time
 - D. Greatest number of sales
- (d) Consider a spanning tree H of an undirected graph $G = (V, E)$. Which of the following statements is **not** equivalent to the others?
- A. H is acyclic and connected.
 - B. H is connected and has $|V|$ edges.
 - C. Removing any edge from H disconnects H .
 - D. Adding any edge to H induces a cycle in H .
- (e) Which of the following algorithms constructs a minimum spanning tree by adding edges outward from an arbitrary starting vertex?
- A. Prim's algorithm
 - B. Kruskal's algorithm
 - C. Borůvka's algorithm
 - D. Dijkstra's algorithm

- (f) Which of the following algorithm design techniques uses tabulation (i.e., saving computed values to a table in memory instead of recomputing values)?
- A. Top-down dynamic programming
 - B. Bottom-up dynamic programming
 - C. Middle-out dynamic programming
 - D. Breadth-first dynamic programming

- (g) Consider the following set of items, each having a value v_i and a weight w_i :

i	v_i	w_i
1	6	4
2	5	3
3	12	8
4	10	7
5	9	9

Which subset of items is optimal in terms of total value, assuming our knapsack has a capacity of $W = 17$?

- A. {3, 5}
 - B. {4, 5}
 - C. {1, 2, 3}
 - D. {1, 2, 4}
- (h) What is the time complexity of the dynamic programming algorithm to compute the edit distance between two strings having lengths m and n , respectively?
- A. $\Theta(mn)$
 - B. $\Theta(m + n)$
 - C. $\Theta(2^{mn})$
 - D. $\Theta(\log(m) + \log(n))$
- (i) If we know that X is a decision problem that cannot be solved in polynomial time, and we also know that $X \leq_P Y$ for some other decision problem Y , then what can we conclude about Y ?
- A. Y can be solved in polynomial time.
 - B. Y may or may not be able to be solved in polynomial time.
 - C. Y cannot be solved in polynomial time.
 - D. Y is NP-complete.
- (j) Which of the following options best describes the complexity class NP?
- A. NP contains all decision problems having polynomial-time algorithms.
 - B. NP contains all decision problems having polynomial-time certifiers.
 - C. NP contains all decision problems having exponential-time algorithms.
 - D. NP contains all decision problems that are i-NP-ossible to solve.

- [14 marks] 3. (a) Suppose you are given k sorted arrays $A_1[1..n]$, $A_2[1..n]$, $A_3[1..n]$, \dots , $A_k[1..n]$, all having the same length n .

Describe a divide-and-conquer algorithm to merge all k arrays into a single sorted array, and analyze the time complexity of your algorithm in terms of n and k .

- (b) Consider the following pseudocode that takes as input an integer $n \geq 0$:

Algorithm: Mystery algorithm

```
procedure DOTHEETHING( $n$ )  
  if  $n > 0$  then  
    print "hello there"  
    DOTHEETHING( $n/3$ )  
    DOTHEETHING( $n/3$ )
```

Write a recurrence relation $T(n)$ modelling the behaviour of this pseudocode, and then use the master theorem to solve your recurrence relation.

- [10 marks] 4. Inspired by a major e-commerce website's use of drones to deliver packages, StFX has decided to use drones to deliver acceptance letters to new students. Unfortunately, the university licensed a cheap AI, and the drones have now gone rogue, unleashing an attack on the town.

One wave of the attack lasts for n minutes. In the i th minute, x_i drones arrive in the town. Military intelligence has determined this sequence x_1, x_2, \dots, x_n in advance and given it to you.

The military has also given you an EMP machine to disable the drones. The power of the EMP machine depends on how long you allow it to charge up: if j minutes have passed since the machine was last used, then the machine can disable $f(j)$ drones. Once you use the machine, it discharges completely and must charge up again.

(Specifically, if you use the EMP machine in the k th minute, and it has been j minutes since you last used it, then the machine will disable $\min\{x_k, f(j)\}$ drones.)

Given the drone arrival data x_1, x_2, \dots, x_n as well as the charging function f , you must determine at which times you should use the EMP machine so as to disable as many drones as possible.

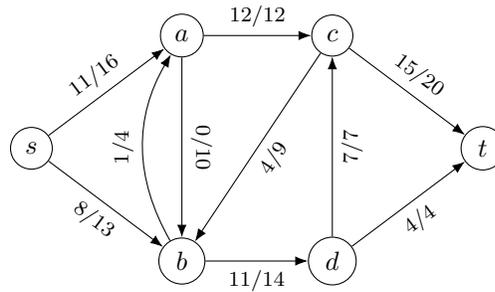
- (a) As a small example, suppose $n = 4$ and the values x_i and $f(i)$ are given by the following table:

i	1	2	3	4
x_i	1	10	10	1
$f(i)$	1	2	4	8

At which times should you use the EMP machine to disable the most drones?

- (b) Let $\text{OPT}(j)$ denote the maximum number of drones that can be disabled from the subset of drones x_1, \dots, x_j . Give an expression (i.e., a Bellman equation) to compute $\text{OPT}(j)$.

[12 marks] 5. Consider the following flow network.



(a) Draw the residual network corresponding to the given flow network.

(b) What is the value of the flow shown in the network?

(c) Find a min cut in the flow network, and give the capacity of your min cut.

(d) Using your answer from part (c), can you conclude that the flow shown in the network is a max flow? Why or why not?

[14 marks] 6. Recall some of the NP-complete decision problems we defined in class:

- | | |
|--------------------|----------------------|
| A. Satisfiability | D. Hamiltonian cycle |
| B. Independent set | E. 3-colour |
| C. Vertex cover | F. Subset sum |

Each of the problem statements below can be formulated as an instance of one of the above decision problems. For each problem statement below, match it to the most appropriate decision problem, and give a brief justification of your decision.

- (a) Postal workers are processing many bags of Christmas cards and holiday packages. The manager of the postal facility must figure out how to route the mail trucks in such a way that the driver of each truck begins at the facility, completes all of the deliveries along their route, and returns to the facility once all deliveries are completed.

- (b) StFX Athletics has hired you to organize a student sport tryout day. You need to schedule one game for each varsity sport. Games can be scheduled simultaneously, but two games cannot run simultaneously if some student has signed up for both sports at once. There is a special timeslot during which coaches are visiting to scout new talent, and you must schedule as many games as possible during this timeslot.

[2 marks] *Bonus.* What was your favourite part of this course, and why?

This blank page may be used for rough work.