

St. Francis Xavier University
Department of Computer Science
CSCI 355: Algorithm Design and Analysis
Midterm Examination
November 1, 2024
1:30pm–2:20pm

Student Name: _____

Email Address: _____

Instructor: T. J. Smith (Section 10)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Question	Marks	Score
1	5	
2	7	
3	6	
4	7	
Total	25	

Signature: _____

Multiple Choice

- [5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.
- (a) Which of the following statements about the Gale–Shapley algorithm is **false**?
- A. The algorithm terminates after at most n^2 steps, where n is the preference list size.
 - B. The match produced by the algorithm is optimal for all parties involved.
 - C. The match produced by the algorithm is guaranteed to be stable.
 - D. The algorithm produces a matching for every input instance of preference lists.
- (b) Suppose we have an algorithm that takes an array of size n as input. For each additional element in the input array, the amount of work done by the algorithm doubles. What is the runtime of such an algorithm?
- A. $O(n)$.
 - B. $O(2n)$.
 - C. $O(n^2)$.
 - D. $O(2^n)$.
- (c) Which of the following heuristics is optimal for the variant of the scheduling problem whose objective is to minimize lateness?
- A. Earliest start time first
 - B. Earliest finish time first
 - C. Earliest deadline first
 - D. Shortest processing time
- (d) Which of the following algorithms constructs a minimum spanning tree by sorting the edge set and adding edges in order from least cost to greatest cost?
- A. Prim’s algorithm.
 - B. Kruskal’s algorithm.
 - C. Borůvka’s algorithm.
 - D. Dijkstra’s algorithm.
- (e) Suppose we run a deterministic comparison-based sorting algorithm on an array of size n . What is the **lower bound** on the runtime of such an algorithm, assuming comparisons take constant time?
- A. $\Omega(1)$.
 - B. $\Omega(n)$.
 - C. $\Omega(n \log(n))$.
 - D. $\Omega(n^2)$.

Short Answer

- [7 marks] 2. (a) Consider the following pseudocode. State the runtime of this algorithm in terms of Big-O notation, assuming n is the input size. Also give a brief (1-2 sentence) justification of your answer. You can assume that assignments and arithmetic operations take $O(1)$ time. Note that you should give the tightest upper bound possible as your answer.

Algorithm: Mystery algorithm

```

s ← 0
for 1 ≤ i ≤ n do
    s ← s + 2i
    k ← n
    while k ≥ 1 do
        s ← s + 4k
        k ← k/2
    
```

- (b) For each of the following statements about Big-O notation, indicate whether the statement is true or false. If the statement is false, write the correct statement.

Statement	True/false?	Correct statement (if false)
If $f_1 \in O(g_1)$ and $f_2 \in O(g_2)$, then $f_1 + f_2 \in O(g_1 + g_2)$.		
If $f_1 \in O(g_1)$ and $f_2 \in O(g_2)$, then $f_1 \cdot f_2 \in O(g_1 \cdot g_2)$.		
If $f \in O(g)$ and $h \in O(g)$, then $f \in O(h)$.		

- [6 marks] 3. A team of professional robbers from out of town are planning to steal a shipment of X-rings from the university. In order for the robbers to plan their escape route, they have obtained a map of Antigonish in the form of a directed graph $G = (V, E)$, where vertices represent town buildings and edges represent streets. (G is a directed graph because the townsfolk are nosy and they would notice the robbers going up and down the same street.)

One vertex, x , denotes the university. A subset of vertices $R \subseteq V$ designates the exits out of town. Each directed edge $e \in E$ has a non-negative weight w_e indicating the time it takes to go down that street.

- (a) Describe an algorithm to compute the shortest time needed for the robbers to exit the town, starting from the university. Your algorithm will receive as input the graph G , a weight value w_e for each edge e , the distinguished vertex x , and the subset of vertices R .

- (b) What is the runtime of your algorithm from part (a) in terms of the size of the input graph $G = (V, E)$? You do not need to give a formal proof, but you should give a brief (1–2 sentence) justification of your answer.

[7 marks] 4. (a) In lecture, we studied both the quicksort and the quickselect algorithms. Give one similarity and one difference between these two algorithms.

(b) Consider the following array A :

[27, 46, 28, 12, 18, 30, 7, 28, 21, 18, 51, 51, 8, 15, 49, 26, 36, 16, 44, 33, 45, 30, 6, 15, 9]

Using the median-of-medians technique, find an appropriate pivot element to partition A .

The median-of-medians pivot element is _____ .

[1 mark] *Bonus.* What were the surnames of the five people who introduced the median-of-medians technique?

This blank page may be used for rough work.