Assignment Regulations.

- This assignment must be completed individually.

- Please include your full name and email address on your submission.

- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[7 marks] 1. (a) Consider the context-free language

$$L_1 = \{\mathtt{a}^{2n}\mathtt{b}^{3n} \mid n \geq 0\}.$$

Construct a pushdown automaton that recognizes this language.

(b) Consider the context-free language

$$L_2 = \{\mathtt{a}^{2i}\mathtt{b}^{i}\mathtt{c}^{3j}\mathtt{d}^{j} \mid i \geq 1, j \geq 1\}.$$

Give a context-free grammar that generates this language.

[6 marks] 2. Consider the following grammar:

$$S \to \mathtt{a}S\mathtt{a} \mid B$$
$$B \to \mathtt{bb}C \mid \mathtt{bb}$$
$$C \to \mathtt{c}C \mid \epsilon$$

Convert this grammar to an equivalent grammar in Chomsky normal form.

[7 marks] 3. Consider the following context-free grammar G, where $V = \{S, A, B, C\}$, $\Sigma_G = \{\mathtt{a}, \mathtt{b}\}$, the start nonterminal is S, and the rule set contains the following rules:

$$S \to SB \mid AB$$
$$A \to CC$$
$$B \to \mathtt{b}$$
$$C \to \mathtt{a}$$

Convert G to an equivalent pushdown automaton $\mathcal{N}$. You do not need to draw the pushdown automaton, though you can if you like; you just need to give each component of the tuple $\mathcal{N} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

[5 marks] 4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Using the pumping lemma for context-free languages, prove that the following language is not context-free:

$$L = \{\mathtt{a}^{i+1}\mathtt{b}^{2j}\mathtt{c}^{2j}\mathtt{d}^{2j} \mid i \geq 0, j \geq 0\}.$$