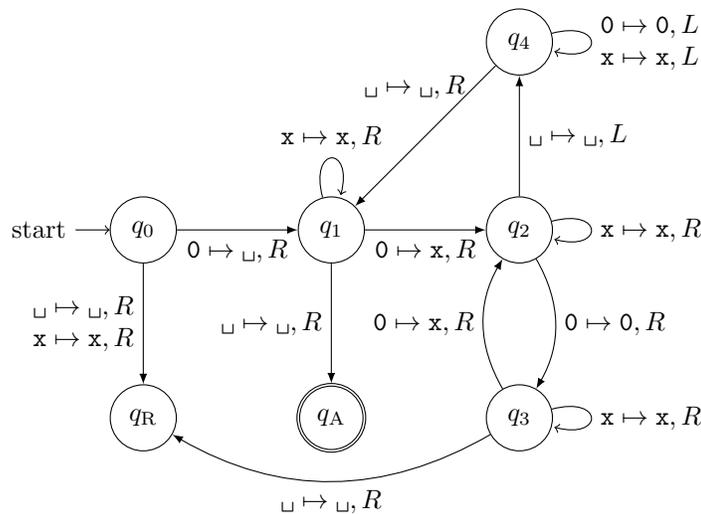


St. Francis Xavier University
Department of Computer Science
CSCI 356: Theory of Computing
Assignment 4
Due December 5, 2025 at 11:30am

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Consider the following Turing machine \mathcal{M} :



- (a) Give the sequence of configurations that \mathcal{M} enters on the input word 00. Does \mathcal{M} accept 00?
- (b) Give the sequence of configurations that \mathcal{M} enters on the input word 000. Does \mathcal{M} accept 000?
- (c) Give two other examples of input words accepted by \mathcal{M} . (You do not have to give the entire sequence of configurations for each input word.)
- (d) Based on your findings from parts (a)–(c), what language is recognized by \mathcal{M} ?

[6 marks] 2. Describe a deterministic single-tape Turing machine that decides the following language L over the alphabet $\Sigma = \{a, b\}$:

$$L = \{w \in \Sigma^* \mid |w|_a \leq |w|_b \leq 2 \cdot |w|_a\}.$$

Here, the notation $|w|_a$ denotes the number of occurrences of the symbol a in the word w .

Note. You do not need to construct the Turing machine explicitly; you need only give a set of steps describing how some Turing machine \mathcal{M} would decide the language L .

- [4 marks] 3. Consider a variant of the Turing machine model where the transition function is defined as follows:

$$\delta : (Q \setminus \{q_{\text{accept}}, q_{\text{reject}}\}) \times \Gamma \rightarrow Q \times \Gamma \times \{R, N\}.$$

This variant machine is only able to move its input head rightward by one cell, or stay where it is (i.e., make “no move”).

Does this variant machine recognize the same class of languages as an ordinary Turing machine? If so, explain why. If not, explain which class of languages it recognizes and justify your reasoning.

- [7 marks] 4. Consider the following decision problem:

$SUB_{\text{DFA}} = \{\langle \mathcal{B}, w \rangle \mid \mathcal{B} \text{ is a deterministic finite automaton and some word in } L(\mathcal{B}) \text{ contains } w \text{ as a subword}\}.$

(A word w is a *subword* of another word v if $v = \Sigma^*w\Sigma^*$.)

Prove that SUB_{DFA} is decidable by giving a decision algorithm.

Hint. You may find the fact that the class DFA is closed under the operation of intersection to be useful.

- [5 marks] 5. Choose your favourite topic from the course, and write a multiple-choice style question with one correct answer and 3–4 plausible-but-incorrect answers that tests a concept or notion related to that topic.

For inspiration, consider the multiple-choice style questions you saw on the midterm and practice midterm exams.