St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Midterm Examination
October 30, 2024
12:30pm–1:20pm

Student Name: _____

Email Address: _____

Instructor: T. J. Smith (Section 10)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: _____

Question	Marks	Score
1	5	
2	8	
3	7	
4	5	
Total	25	

Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Let L be the language corresponding to the regular expression $((\texttt{00})^*\texttt{11} + \texttt{01})^*$. Which of the following words is in L?

A. `0001`.
B. `011100`.
C. `01110011`.
D. `11110110`.
E. All of the above words are in L.

(b) Given a deterministic finite automaton $\mathcal{D}$ recognizing a language L, what must we do to construct a deterministic finite automaton $\mathcal{D}'$ recognizing the complement language $\overline{L}$?

A. For each transition of $\mathcal{D}$, reverse the direction of the transition in $\mathcal{D}'$.
B. Make all final states of $\mathcal{D}$ non-final states in $\mathcal{D}'$, and make all non-final states of $\mathcal{D}$ final states in $\mathcal{D}'$.
C. Merge all final states of $\mathcal{D}$ into a single final state in $\mathcal{D}'$, then make this the initial state of $\mathcal{D}'$.
D. We cannot construct a deterministic finite automaton that recognizes $\overline{L}$.

(c) Which of the following statements is **false**?

A. Every finite language is regular.
B. Every infinite language is regular.
C. Every non-regular language is infinite.
D. Every regular language is context-free.

(d) Which of the following context-free grammars is in Chomsky normal form?

A. $S \to TT \mid \texttt{a}$
$T \to \texttt{b}$
B. $S \to TT \mid \texttt{b}$
$T \to T\texttt{b}T \mid \texttt{a}$
C. $S \to \texttt{a}T \mid TT$
$T \to \texttt{b}$
D. $S \to ST$
$T \to \texttt{a} \mid \texttt{bb}$
E. All of the above grammars are in Chomsky normal form.

(e) Which of the following statements is **true**?

A. There exists a language that is recognized by some nondeterministic finite automaton, but not by any deterministic finite automaton.
B. There exists a language that is recognized by some deterministic finite automaton, but not by any nondeterministic finite automaton.
C. There exists a language that is recognized by some nondeterministic pushdown automaton, but not by any deterministic pushdown automaton.
D. There exists a language that is recognized by some deterministic pushdown automaton, but not by any nondeterministic pushdown automaton.

Short Answer

[8 marks] 2. (a) Using each of the following example sets Eg_{x} as a guide, give a regular expression that would match the pattern demonstrated by each of the words in the example set.

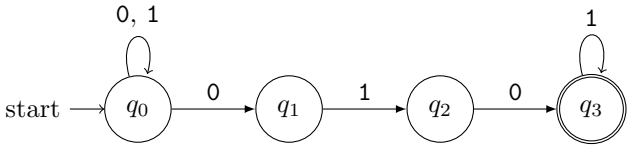
Your regular expressions must be as general as possible (meaning that you can't just take the union of each of the individual words).

i. $\Sigma = \{\texttt{o}, \texttt{h}\}$, $Eg_{\text{ghost}} = \{\texttt{ooooh}, \texttt{ooohohoh}, \texttt{oohooooooh}\}$

ii. $\Sigma = \{\texttt{a}, \texttt{h}, \texttt{m}, \texttt{u}\}$, $Eg_{\text{villain}} = \{\texttt{haha}, \texttt{muaha}, \texttt{muahahahahaha}\}$

iii. $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{i}, \texttt{n}, \texttt{r}, \texttt{s}\}$, $Eg_{\text{zombie}} = \{\texttt{brains}, \texttt{braaaaains}, \texttt{braaaaaaaaaains}\}$

(b) Consider the following nondeterministic finite automaton:



Suppose we use the subset construction to convert this nondeterministic finite automaton into an equivalent deterministic finite automaton. Give the transition table that results from applying the subset construction to the nondeterministic finite automaton.

(You do not need to draw the deterministic finite automaton; only give its transition table.)

	0	1
q_0	$\{q_0, q_1\}$	q_0
q_1	—	q_2
q_2	q_3	—
q_3	—	q_3

[7 marks] 3. (a) Let $\Sigma = \{\mathtt{b}, \mathtt{c}\}$. Consider the context-free language

$$L_1 = \{\mathtt{b}^{2i}\mathtt{c}^{3j+1} \mid j \geq i \geq 0\}.$$

i. Give a context-free grammar that generates L_1.

ii. Using your context-free grammar, give a derivation for the word $\mathtt{b}^2\mathtt{c}^4 \in L_1$.

(b) Let $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$. Consider the context-free language

$$L_2 = \{w\mathtt{z}w^{\mathrm{R}} \mid w \in \{\mathtt{x}, \mathtt{y}\}^*\}.$$

Here, the notation w^{R} denotes the reversal of the word w. For example, if $w = \mathtt{xyy}$, then $w^{\mathrm{R}} = \mathtt{yyx}$. Construct a pushdown automaton that recognizes L_2.

[5 marks] 4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L = \{\mathtt{d}^{2i+1}\mathtt{c}^{2j+1}\mathtt{b}^{k+2}\mathtt{a}^{\ell+3} \mid i \geq j \geq 0, k \geq 0, \ell \geq 0\}.$$

This language is nonregular, and we can use the pumping lemma to prove nonregularity. Below, the first and last parts of the proof are given to you. Fill in the rest of the proof.

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Assume by way of contradiction that L is regular, and let p denote the pumping constant given by the pumping lemma. We choose the word $w =$ _____. Clearly, $w \in L$ and $|w| \geq p$. Then, there exists a decomposition $w = xyz$ satisfying the three conditions of the pumping lemma.

Thus, not all conditions of the pumping lemma have been satisfied, and we have reached a contradiction. As a consequence, the language L cannot be regular.

This blank page may be used for rough work.