St. Francis Xavier University
Department of Computer Science

CSCI 544: Computational Logic
Assignment 1
Due October 15, 2025 at 9:30am

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[6 marks] 1. (a) Translate each of the following English sentences into the syntax of propositional logic using the indicated letters. For each letter, indicate explicitly the statement it stands for.

i. Jesse eats chicken-flavoured cat treats only if Darla didn't eat tuna-flavoured cat treats. (c, t)

ii. Winning the Turing award is sufficient for you to get a job in the StFX Department of Computer Science. (w, j)

iii. Society must eliminate narrow-minded fundamentalist viewpoints or accept an ethical absolutism imposed by a small but vocal group. (e, a)

(b) Translate the following Kantian argument into the syntax of propositional logic, and then prove that the argument is valid.

Premise 1. If we had a knowledge of God, then our will would be irresistibly attracted to do right.

Premise 2. We would have no free will if our will were irresistably attracted to do right.

Conclusion. Us having a knowledge of God means we have no free will.

[4 marks] 2. Two common methods of backing up data make use of *redundancy* and *striping*: we can back up a file by splitting it into chunks, storing each of these chunks on separate drives, and maintaining multiple copies of a *parity value* on each drive to verify whether the reassembled chunks still make up the original file. What happens when one of these chunks is lost, though? Is the original file lost?

Suppose we have a file made up of three chunks: A, B, and C. We can verify that the three chunks make up the original file by taking the exclusive-OR of the chunks and comparing it to the parity value P; that is, we verify that $A \oplus B \oplus C = P$.

(a) Suppose we lose the B chunk. Show how we can still recover the original file using only the chunks A and C together with the parity value P.

Hint. You may find the following properties of the exclusive-OR operation helpful: $A \oplus 0 = A$ for all A (identity) and $A \oplus A = 0$ for all A (self-inverse).

(b) Now, suppose we lose *both* the A and B chunks. Can we still recover the original file using only the remaining C chunk and the parity value P? Why or why not?

[9 marks] 3. Given a set C of logical connectives, we say that the set is *adequate* if all other logical connectives can be expressed as some combination of only the connectives in C.

(a) Show that we can express the connective $\vee$ in terms of the set of connectives $\{\neg, \wedge\}$.

(b) Explain why the set $\{\neg, \wedge, \vee\}$ is an adequate set of connectives.

Hint. Suppose we have a truth table. Can we convert the truth table to a propositional formula sharing the same truth values using only these three connectives?

(c) Explain why the set $\{\wedge, \Rightarrow\}$ is *not* an adequate set of connectives.

Hint. Suppose we have an interpretation assigning true to every formula. What will the truth value be of a formula using only $\wedge$ and $\Rightarrow$, and what does this imply?

[6 marks] 4. Using the method of semantic tableaux, determine whether each of the following formulas are valid. If a formula is not valid, give an example of an interpretation demonstrating this. (Remember that a formula A is valid if and only if $\neg A$ is unsatisfiable.)

(a) $A = (p \wedge q) \Rightarrow (p \vee r)$.

(b) $B = (p \vee \neg(q \wedge r)) \Rightarrow ((p \Leftrightarrow r) \vee q)$.

[10 marks] 5. Prove the validity of each of the following sequents using natural deduction.

(a) $(p \wedge q) \wedge r \vdash p \wedge (q \wedge r)$.

(b) $p \Rightarrow q, r \Rightarrow s \vdash (p \vee r) \Rightarrow (q \vee s)$

[5 marks] 6. Using natural deduction, prove the following:

$$(p \Rightarrow q) \Rightarrow ((\neg p \Rightarrow q) \Rightarrow q).$$

Hint. You may need to use the *Law of Excluded Middle* in your proof: $p \vee \neg p$.