St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Assignment 3
Due November 20, 2026 at 11:30am

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[6 marks] 1. Consider the following grammar:

$$S \to \mathtt{a}S\mathtt{a} \mid B$$
$$B \to \mathtt{bb}C \mid \mathtt{bb}$$
$$C \to \mathtt{c}C \mid \epsilon$$

Convert this grammar to an equivalent grammar in Chomsky normal form.

[7 marks] 2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L_1 = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^{2j}\mathtt{d}^{2i} \mid i \geq 1, j \geq 1\}.$$

Construct a pushdown automaton that recognizes the language L_1, then give a trace of the behaviour of your pushdown automaton on the input word `abbccccdd`. Your trace should list, at each step of the computation: the current state of the pushdown automaton, the current stack contents, and the remaining input word symbols.

[7 marks] 3. Consider the following context-free grammar G, where $V = \{R, S, T, X\}$, $\Sigma_G = \{\mathtt{a}, \mathtt{b}\}$, the start nonterminal is R, and the rule set contains the following rules:

$$R \to XRX \mid S$$
$$S \to \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a}$$
$$T \to XTX \mid X \mid \epsilon$$
$$X \to \mathtt{a} \mid \mathtt{b}$$

Convert G to an equivalent pushdown automaton $\mathcal{M}$. You do not need to draw the pushdown automaton, you just need to give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

[5 marks] 4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Using the pumping lemma for context-free languages, prove that the following language is not context-free:

$$L = \{\mathtt{a}^k\mathtt{b}^{2k}\mathtt{c}^i \mid k \geq i \geq 1\}.$$

[7 marks] 5. **Only CSCI 541 students:**

Consider the following context-free grammar G, where $V = \{S, A, B, C\}$ and $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}, \texttt{!}, \texttt{\#}\}$:

$$S \to \texttt{111}A\texttt{3}C \mid A\texttt{!}$$
$$A \to \texttt{111}A\texttt{3} \mid B$$
$$B \to \texttt{2}B\texttt{11} \mid \texttt{\#}$$
$$C \to \texttt{!}$$

This context-free grammar, with seven rules, is overly complicated. Construct an equivalent context-free grammar with *at most five* rules that generates the same language, and give a short explanation of why your grammar is equivalent to the original grammar. You may change the nonterminal alphabet V, but your grammar must use the same terminal alphabet Σ.

[8 marks] 6. **Only CSCI 541 students:**

Ogden's lemma is a strengthening of the pumping lemma for context-free languages. It differs from the pumping lemma in that, if we focus on any p 'marked' symbols in a word, pumping results in a word having between 1 and p 'marked' symbols in it. (The pumping lemma is the special case where all symbols are 'marked'.)

We can state Ogden's lemma in the following way:

> If L is a context-free language, then there exists a pumping length p such that, for every $w \in L$ of length at least p and for every way of 'marking' p or more symbols in w, we can write $w = uvxyz$ such that:
>
> 1. vy contains at least 1 'marked' symbol;
> 2. vxy contains at most p 'marked' symbols; and
> 3. for all $i \geq 0$, $uv^i xy^i z \in L$.

Use Ogden's lemma to prove that the language $L = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k\texttt{d}^\ell \mid i = 0 \text{ or } j = k = \ell\}$ is not context-free.

[10 marks] 7. **Only CSCI 541 students:**

Investigate the class of context-sensitive languages. Define these languages, explain what kinds of models of computation recognize these languages, and compare the properties of this class to those of the class of context-free languages. What are some examples of context-sensitive languages, and how can we prove these languages are context-sensitive? Conversely, are there any examples of non-context-sensitive languages? What applications can you find for context-sensitive languages, particularly in linguistics?

Find and cite at least three non-internet sources to support your answer.