St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Midterm Examination
October 24, 2025
11:30am–12:20pm

Student Name: _____

Email Address: _____

Instructor: T. J. Smith (Section 10)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: _____

Question	Marks	Score
1	5	
2	7	
3	8	
4	5	
Total	25	

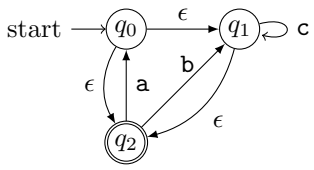
Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Let L be the language matched by the regular expression $(\mathtt{a}+\mathtt{ba}^*\mathtt{b}+\mathtt{bc}^*\mathtt{b})^*$. Which of the following words is **not** in L?

A. `aabcbbb`.
B. `babcbab`.
C. `bcbabcb`.
D. `bcbbba`.

(b) Consider the following nondeterministic finite automaton with ϵ-transitions:



After we convert this to a nondeterministic finite automaton *without* ϵ-transitions, what will be the label of the transition from state q_0 to state q_1?

A. ϵ `+ a`
B. `a + c`
C. `b + c`
D. `c`

(c) Consider the following partial transition table produced after determinizing a nondeterministic finite automaton. What is the missing row of the table?

	a	b	c	d
q_0	q_0	$\{q_1, q_2\}$	—	—
q_1	q_2	—	—	q_1
q_2	—	—	$\{q_0, q_2\}$	q_1
$\{q_0, q_2\}$	q_0	$\{q_1, q_2\}$	$\{q_0, q_2\}$	q_1

A. $\{q_1, q_2\}$ | q_2 — $\{q_0, q_2\}$ q_1
B. $\{q_1, q_2\}$ | q_2 $\{q_1, q_2\}$ $\{q_0, q_2\}$ q_1
C. $\{q_1, q_2\}$ | q_0 $\{q_1, q_2\}$ $\{q_0, q_2\}$ —
D. $\{q_1, q_2\}$ | q_2 — — q_1

(d) Let $L = \{\mathtt{a}^i\mathtt{b}^{2k} \mid 0 \le i \le k\}$. Which of the following grammars generates the language L?

A. $S \to \mathtt{abb}S \mid \mathtt{bb}S \mid \epsilon$
B. $S \to \mathtt{a}S\mathtt{bb} \mid \epsilon$
C. $S \to \mathtt{a}S\mathtt{bb} \mid \mathtt{a}S\mathtt{bbb} \mid \epsilon$
D. $S \to \mathtt{a}S\mathtt{bb} \mid S\mathtt{bb} \mid \epsilon$

(e) Which of the following is **not** a step in the process to convert a context-free grammar to Chomsky normal form?

A. Remove all rules of the form $A \to B$, where $A, B \in V$.
B. Remove all rules of the form $A \to \epsilon$, where $A \neq S$.
C. Remove all rules with more than two nonterminal or terminal symbols on the right-hand side.
D. Remove all rules where some nonterminal symbol appears before a terminal symbol on the right-hand side.

Short Answer

[7 marks] 2. (a) i. Give an example of a language S such that $S = S^*$.

ii. Give an example of a language T such that T^* is finite.

(b) For each of the following languages, give a deterministic or nondeterministic finite automaton recognizing exactly that language.

i. $L_1 = \{w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ contains the subword } \texttt{0101}\}$.
(In other words, $w = x\texttt{0101}y$ for some $x, y \in \{\texttt{0}, \texttt{1}\}^*$.)

ii. $L_2 = \{\epsilon\}$.

(c) Recall the subset construction we used to determinize a nondeterministic finite automaton. If we have a nondeterministic finite automaton with n states recognizing a language L, what is the upper bound on the number of states in the deterministic finite automaton also recognizing L?

[1 mark] *Bonus.* What language L forces the deterministic finite automaton to meet this upper bound?

[8 marks] 3. (a) Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$, and consider the language

$$L = \{w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ is of odd length and its middle symbol is } \mathtt{0}\}.$$

Give a context-free grammar that generates the language L.

(b) Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Consider the following context-free grammar G:

$$S \to A$$
$$A \to \mathtt{a}A\mathtt{b} \mid \mathtt{a}A \mid \mathtt{b}$$

i. Describe the language $L(G)$ generated by this context-free grammar.

ii. Is the context-free grammar G ambiguous? If so, give an example of one word in the language $L(G)$ having two different parse trees, and draw the parse trees. If not, explain why not.

[5 marks] 4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L = \{\mathtt{a}^{2i}\mathtt{b}^{j}\mathtt{c}^{k}\mathtt{d}^{j+1} \mid i \geq 0, j \geq 0, k \geq 0\}.$$

This language is nonregular, and we can use the pumping lemma to prove nonregularity. Below, the first and last parts of the proof are given to you. Fill in the rest of the proof.

—

Assume by way of contradiction that L is regular, and let p denote the pumping constant given by the pumping lemma. We choose the word $w =$ _____. Clearly, $w \in L$ and $|w| \geq p$. Then, there exists a decomposition $w = xyz$ satisfying the three conditions of the pumping lemma.

Thus, not all conditions of the pumping lemma have been satisfied, and we have reached a contradiction. As a consequence, the language L cannot be regular.

This blank page may be used for rough work.