Module 6: Tries

CS 240 - Data Structures and Data Management

Sajed Haque Veronika Irvine Taylor Smith
Based on lecture notes by many previous cs240 instructors

David R. Cheriton School of Computer Science, University of Waterloo

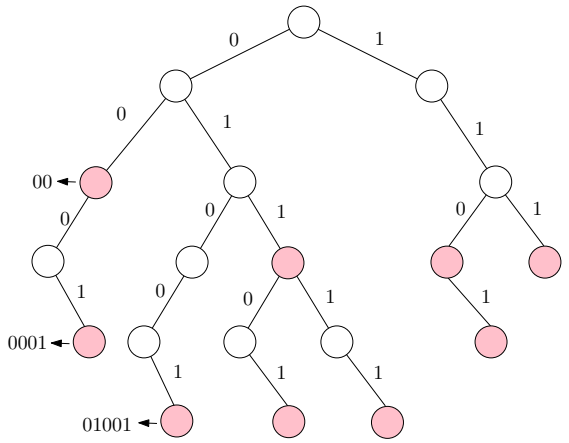
Spring 2017

Tries

- **Trie (Radix Tree)**: A dictionary for binary strings
  - Comes from retrieval, but pronounced "try"
  - A binary tree based on **bitwise comparisons**
  - Similar to **radix sort**: use individual bits, not the whole key
- Structure of trie:
  - A left child corresponds to a 0 bit
  - A right child corresponds to a 1 bit
- Keys can have different number of bits
- Keys are not stored in the trie: a node x is flagged if the path from root to x is a binary string present in the dictionary

Tries

- Example: A trie for
  $S = \{00, 0001, 01001, 011, 01101, 01111, 110, 1101, 111\}$



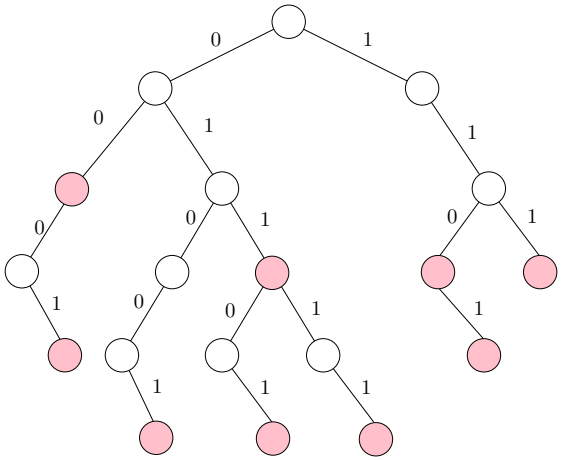
Tries: Search

Search(x):

- start from the root
- take the left link if the current bit in x is 0 and take the right link if it is 1; return failure if the link is missing
- if there are no extra bits in x left and the current node is flagged then - success (x is found)
- recurse

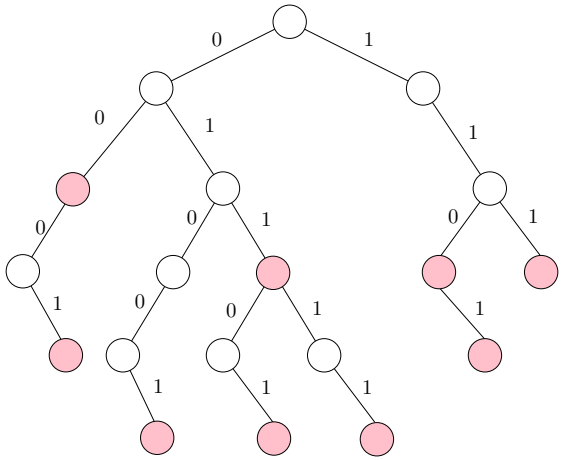
Tries: Search

Example: Search(011)



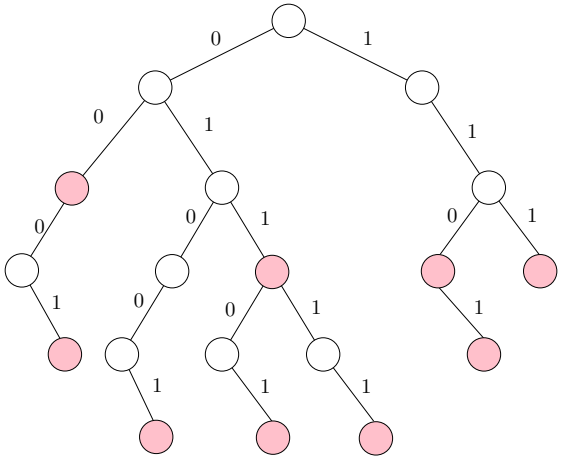
Tries: Search

Example: Search(0101)



Tries: Search

Example: Search(1101)

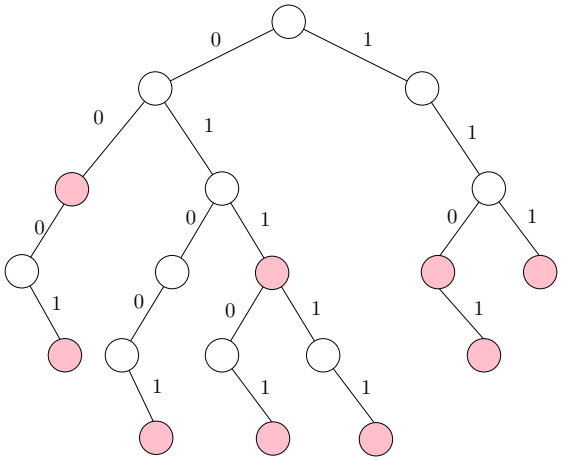


Tries: Insert

- **Insert(x)**
  - Search for x, and suppose we finish at a node v
    Note: x may have extra bits.
  - Expand the trie from the node v by adding necessary nodes that correspond to extra bits of x; flag the last one.

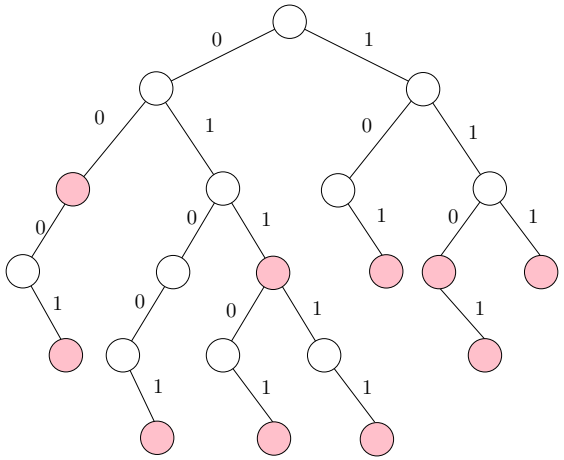
Tries: Insert

Example: Insert(101)



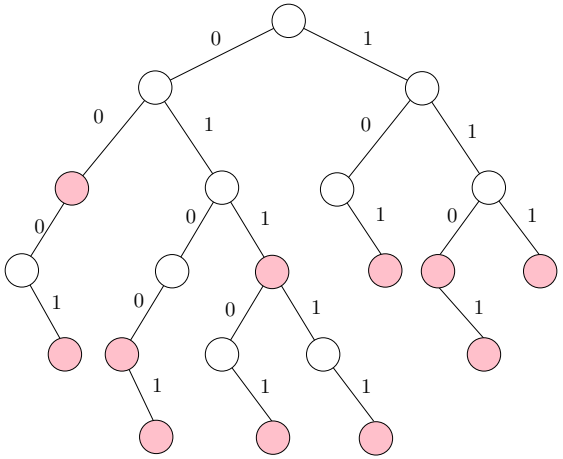
Tries: Insert

Example: Insert(0100)



Tries: Insert

Example: Insert(11101)

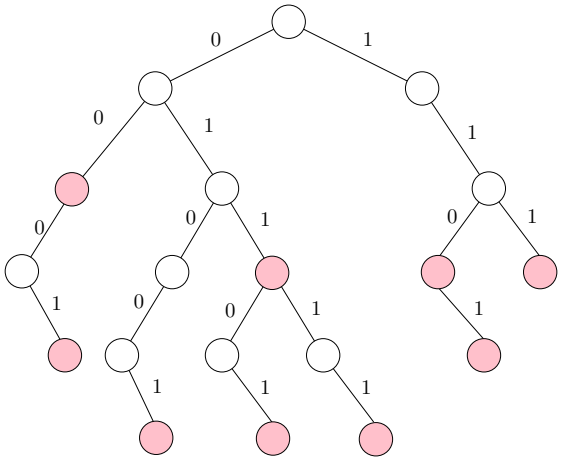


Tries: Delete

- **Delete(x)**
  - Search for x
  - if x found at an internal flagged node, then unflag the node
  - if x found at a leaf v_x, delete the leaf and all **ancestors** of v_x until
    - ⋆ we reach an ancestor that has two children or
    - ⋆ we reach a flagged node

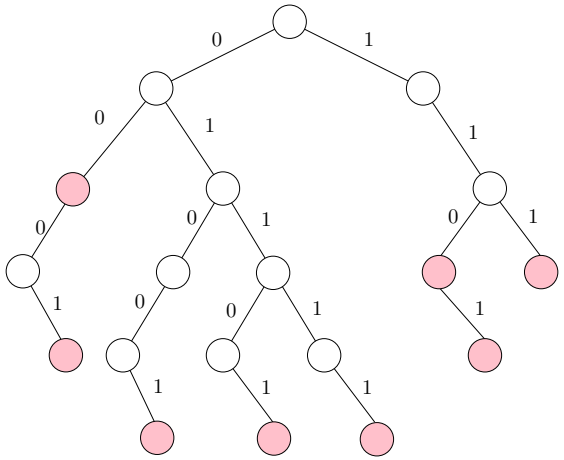
Tries: Delete

Example: Delete(011)



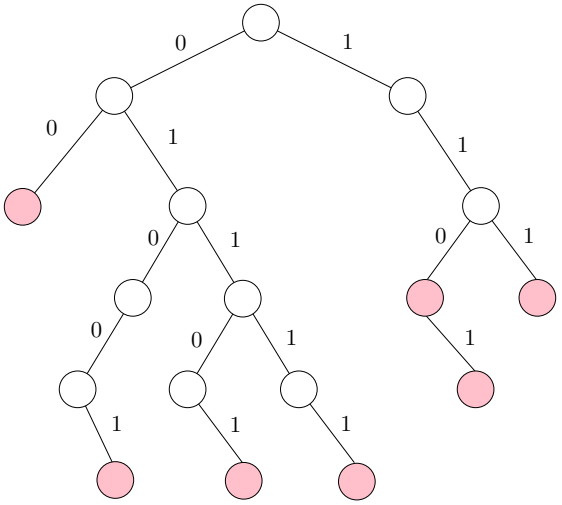
Tries: Delete

Example: Delete(0001)



Tries: Delete

Example: Delete(01001)



Tries: Operations

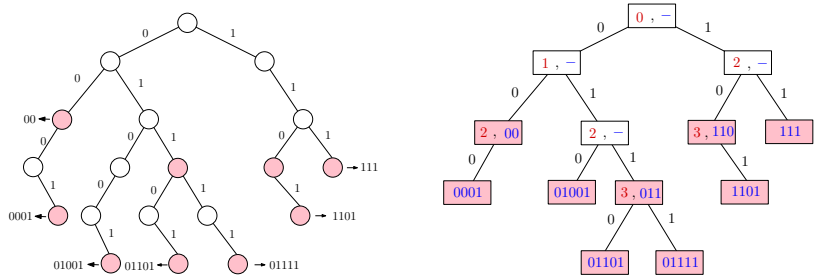
- **Search(x)**
- **Insert(x)**
- **Delete(x)**
- Time Complexity of all operations: $\Theta(|x|)$
  $|x|$: length of binary string x, i.e., the number of bits in x

Compressed Tries (Patricia Tries)

- **Patricia**: Practical Algorithm To Retrieve Information Coded in Alphanumeric
- Introduced by Morrison (1968)
- Reduces **storage requirement**: eliminate unflagged nodes with only one child
- Every path of one-child unflagged nodes is compressed to a single edge
- Each node stores an **index** indicating the next bit to be tested during a search (index= 0 for the first bit, index= 1 for the second bit, etc)
- A compressed trie storing n keys always has at most $n-1$ internal (non-leaf) nodes

Compressed Tries (Patricia Tries)

- Each node stores an **index** indicating the next bit to be tested during a search
- Example: A trie and the equivalent compressed trie

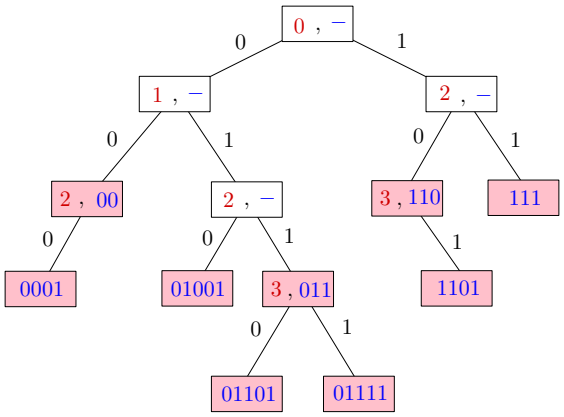


Compressed Tries: Operations

- **Search(x):**
  - Follow the proper path from the root down in the tree to a leaf
  - If search ends in an unflagged node, it is unsuccessful
  - If search ends in a flagged node, we need to check if the key stored is indeed x

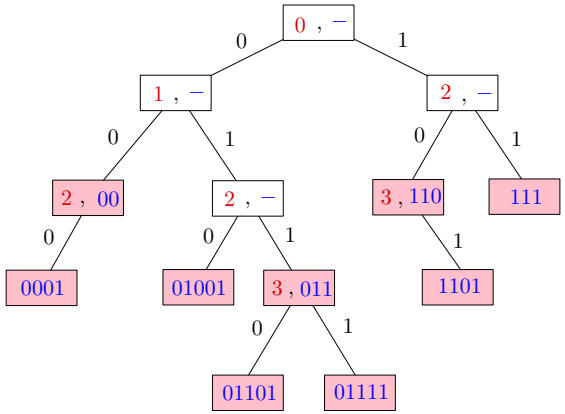
Compressed Tries: Operations

Example: Search(01001)



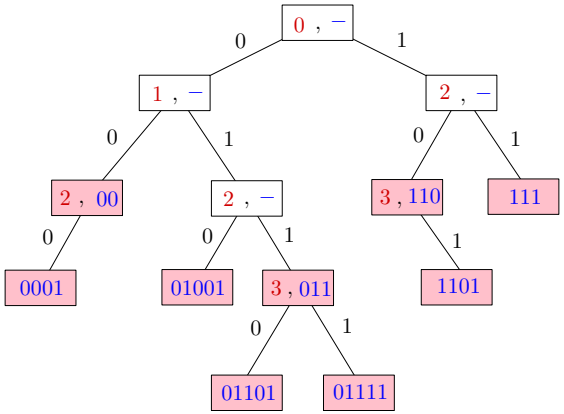
Compressed Tries: Operations

Example: Search(11)



Compressed Tries: Operations

Example: Search(101)

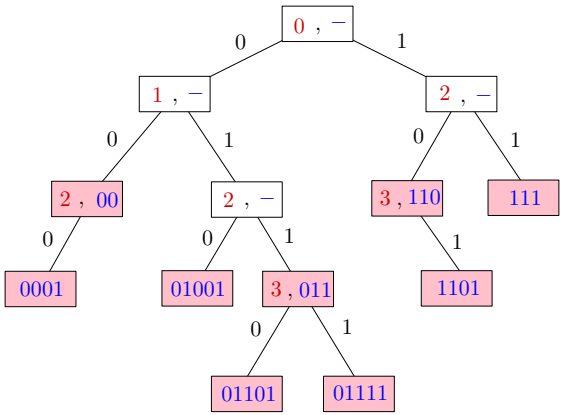


Compressed Tries: Operations

- **Delete(x):**
  - Perform Search(x)
  - if search ends in an internal node, then
    - ⋆ if the node has two children, then unflag the node and delete the key
    - ⋆ else delete the node and make his only child, the child of its parent
  - if search ends in a leaf, then delete the leaf and
  - if its parent is unflagged, then delete the parent

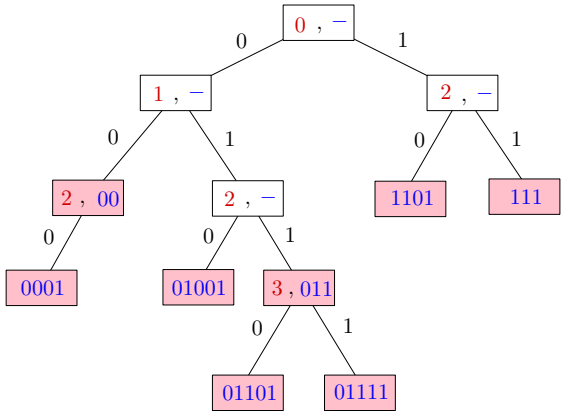
Compressed Tries: Operations

Example: Delete(110)



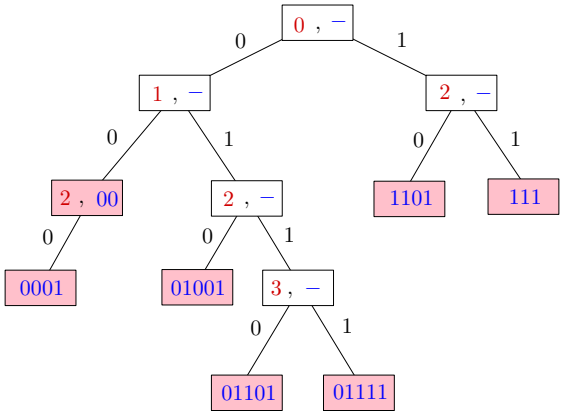
Compressed Tries: Operations

Example: Delete(011)



Compressed Tries: Operations

Example: Delete(01101)

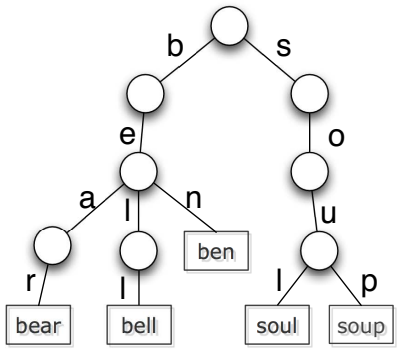


Compressed Tries: Operations

- **Insert(x):**
  - Perform Search(x)
  - If the search ends at a leaf L with key y, compare x against y.
  - If y is a prefix of x, add a child to y containing x.
  - Else, determine the first index i where they disagree and create a **new node** N with index i.
    Insert N along the path from the root to L so that the parent of N has index $< i$ and one child of N is either L or an existing node on the path from the root to L that has index $> i$.
    The other child of N will be a **new leaf node** containing x.
  - If the search ends at an internal node, we find the key corresponding to that internal node and proceed in a similar way to the previous case.

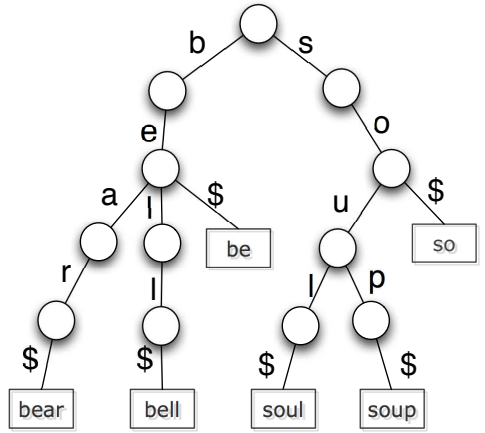
Multiway Tries

- To represent **Strings** over any **fixed alphabet** Σ
- Any node will have at most $|\Sigma|$ children
- Example: A trie holding strings {bear, bell, ben, soul, soup}



Multiway Tries

- Append a special **end-of-word** character, say $, to all keys
- Example: A trie holding strings {bear, bell, be, so, soul, soup}



Multiway Tries

- **Compressed** multi-way tries
- Example: A compressed trie holding strings {bear, bell, be, so, soul, soup}