

Module 9: Tries and String Matching

CS 240 - Data Structures and Data Management

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Based on lecture notes by many previous cs240 instructors

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Pattern Matching

- Search for a string (pattern) in a large body of text
- $T[0..n - 1]$ – The **text** (or **haystack**) being searched within
- $P[0..m - 1]$ – The **pattern** (or **needle**) being searched for
- Strings over **alphabet** Σ
- Return the first i such that

$$P[j] = T[i + j] \quad \text{for } 0 \leq j \leq m - 1$$

- This is the first **occurrence** of P in T
- If P does not **occur** in T , return FAIL
- Applications:
 - ▶ Information Retrieval (text editors, search engines)
 - ▶ Bioinformatics
 - ▶ Data Mining

Pattern Matching

Example:

- $T = \text{"Where is he?"}$
- $P_1 = \text{"he"}$
- $P_2 = \text{"who"}$

Definitions:

- **Substring** $T[i..j]$ $0 \leq i \leq j < n$: a string of length $j - i + 1$ which consists of characters $T[i], \dots, T[j]$ in order
- A **prefix** of T :
a substring $T[0..i]$ of T for some $0 \leq i < n$
- A **suffix** of T :
a substring $T[i..n - 1]$ of T for some $0 \leq i \leq n - 1$

General Idea of Algorithms

Pattern matching algorithms consist of **guesses** and **checks**:

- A **guess** is a position i such that P might start at $T[i]$. Valid guesses (initially) are $0 \leq i \leq n - m$.
- A **check** of a guess is a single position j with $0 \leq j < m$ where we compare $T[i + j]$ to $P[j]$. We must perform m checks of a single **correct** guess, but may make (many) fewer checks of an **incorrect** guess.

We will diagram a single run of any pattern matching algorithm by a matrix of checks, where each row represents a single guess.

Brute-force Algorithm

Idea: Check every possible guess.

```
BruteforcePM( $T[0..n-1]$ ,  $P[0..m-1]$ )
T: String of length  $n$  (text), P: String of length  $m$  (pattern)
1.   for  $i \leftarrow 0$  to  $n - m$  do
2.        $match \leftarrow true$ 
3.        $j \leftarrow 0$ 
4.       while  $j < m$  and  $match$  do
5.           if  $T[i + j] = P[j]$  then
6.                $j \leftarrow j + 1$ 
7.           else
8.                $match \leftarrow false$ 
9.       if  $match$  then
10.          return  $i$ 
11.  return FAIL
```

Example

- Example: $T = \text{abbbababbab}$, $P = \text{abba}$

	a	b	b	b	a	b	a	b	b	a	b
a	b	b	a								
	a										
		a									
			a								
				a	b	b					
					a						
						a	b	b	a		

- What is the worst possible input?
 $P = a^{m-1}b$, $T = a^n$
- Worst case performance $\Theta((n - m + 1)m)$
- $m \leq n/2 \Rightarrow \Theta(mn)$

Pattern Matching

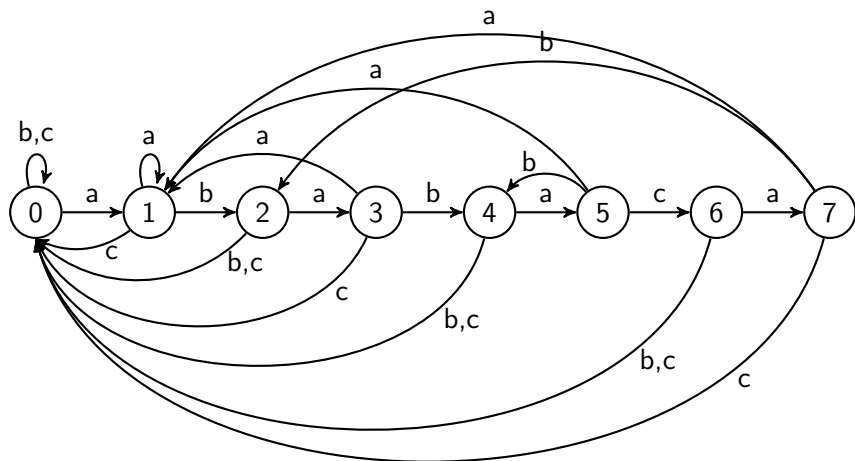
More sophisticated algorithms

- Deterministic finite automata (**DFA**)
- **KMP**, **Boyer-Moore** and **Rabin-Karp**
- Do extra **preprocessing** on the pattern P
- We **eliminate guesses** based on completed matches and mismatches.

String matching with finite automata

There is a string-matching automaton for every pattern P . It is constructed from the pattern in a preprocessing step before it can be used to search the text string.

Example: Automaton for the pattern $P = ababaca$



String matching with finite automata

Let P the pattern to search, of length m . Then

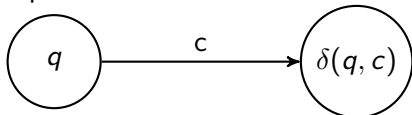
- the *states* of the automaton are $0, \dots, m$
- the *transition function* δ of the automaton is defined as follows, for a state q and a character c in Σ :

$$\delta(q, c) = \ell(P[0..q-1]c),$$

where

- $P[0..q-1]c$ is the concatenation of $P[0..q-1]$ and c
- for a string s , $\ell(s) \in \{0, \dots, m\}$ is the length of the longest prefix of P that is also a suffix of s .

Graphically, this corresponds to



String matching with finite automata

Let T be the text string of length n ,
 P the pattern to search of length m and
 δ the transition function of a finite automaton for pattern P .

FINITE-AUTOMATON-MATCHER(T, δ, m)

$n \leftarrow \text{length}[T]$

$q \leftarrow 0$

for $i \leftarrow 0$ to $n - 1$ do

$q \leftarrow \delta(q, T[i])$

 if $q = m$

 then print "Pattern occurs with shift" $i - (m - 1)$

Idea of proof: the state after reading $T[i]$ is $\ell(T[0..i])$.

String matching with finite automata

- Matching time on a text string of length n is $\Theta(n)$
- This does not include the preprocessing time required to compute the transition function δ . There exists an algorithm with $O(m|\Sigma|)$ preprocessing time.
- Altogether, we can find all occurrences of a length- m pattern in a length- n text over a finite alphabet Σ with $O(m|\Sigma|)$ preprocessing time and $\Theta(n)$ matching time.

KMP Algorithm

- Knuth-Morris-Pratt algorithm (1977)
- Compares the pattern to the text in **left-to-right**
- **Shifts** the pattern more **intelligently** than the brute-force algorithm
- When a mismatch occurs, how much can we shift the pattern (reusing knowledge from previous matches)?

$T =$ a b c d c a b c ? ? ?

a	b	c	d	c	a	b	a			
					a	b	c	d	c	a

- **KMP Answer:** this depends on the largest **prefix** of $P[0..j]$ that is a **suffix** of $P[1..j]$

KMP Failure Array

T: a b b c a b c d ...
P: a b b c a b a a

what next slide would match with the text?

KMP Failure Array

T: a b b c a b c d ...
P: a b b c a b a a
✗ a b b c a b a a

KMP Failure Array

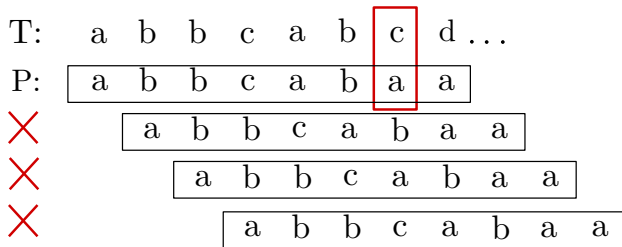
T: a b b c a b c d ...

P: a b b c a b a a

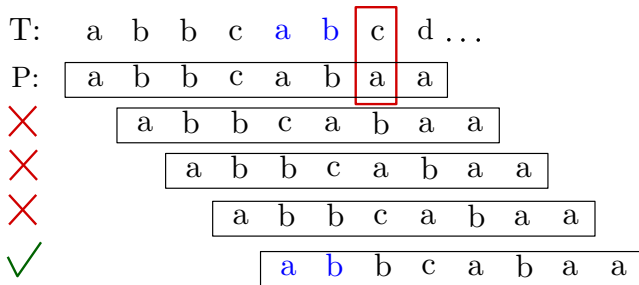
✗ a b b c a b a a

✗ a b b c a b a a

KMP Failure Array



KMP Failure Array



KMP Failure Array

Suppose we have a match up to position $T[i - 1] = P[j - 1]$, but not at the next position.

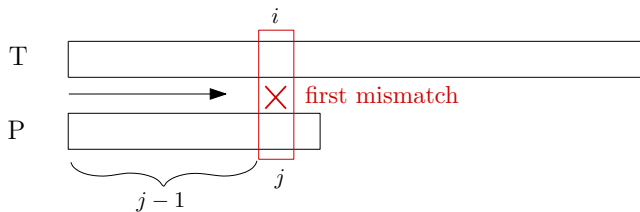
Define $F[j - 1]$ as the index we will have to check in P , after we bring the pattern to its next possible position (previous example: $j = 6$, $F[5] = 2$).

This can be computed by trying all sliding positions until finding the first one matching the text (as in previous example). We can do better:

- any possible sliding position corresponds to a prefix of $P[0..j - 1]$ that is also a *strict* suffix of it = a suffix of $P[1..j - 1]$
- the next possible sliding position corresponds to the **largest** such prefix / suffix
- we let $F[j - 1]$ be the length of this prefix / suffix.

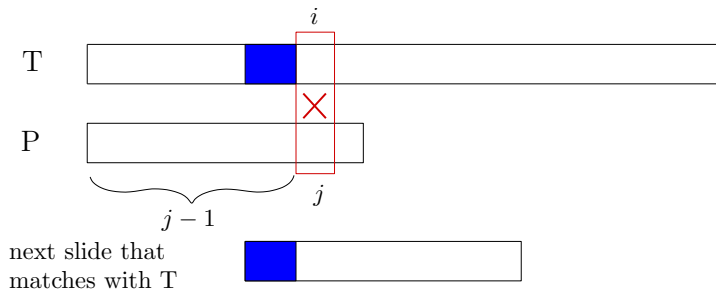
KMP Failure Array

Schematically:



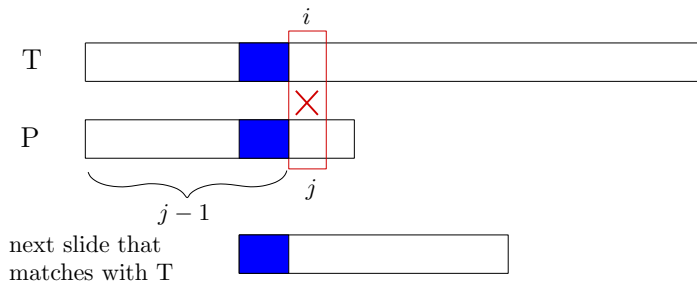
KMP Failure Array

Schematically:



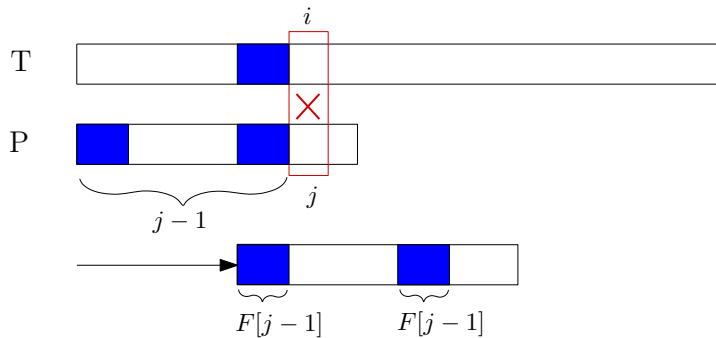
KMP Failure Array

Schematically:



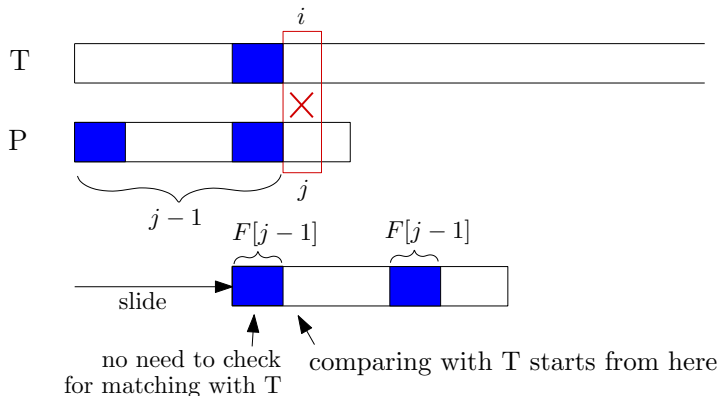
KMP Failure Array

Schematically:



KMP Failure Array

Schematically:



KMP Failure Array

- $F[0] = 0$
- $F[j]$, for $j > 0$, is the length of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Consider $P = \text{abacaba}$

j	$P[1..j]$	P	$F[j]$
0	—	abacaba	0
1	b	abacaba	0
2	ba	abacaba	1
3	bac	abacaba	0
4	baca	abacaba	1
5	bacab	abacaba	2
6	bacaba	abacaba	3

Computing the Failure Array

```
failureArray(P)
P: String of length m (pattern)
1.    $F[0] \leftarrow 0$ 
2.    $i \leftarrow 1$ 
3.    $j \leftarrow 0$ 
4.   while  $i < m$  do
5.       if  $P[i] = P[j]$  then
6.            $F[i] \leftarrow j + 1$ 
7.            $i \leftarrow i + 1$ 
8.            $j \leftarrow j + 1$ 
9.       else if  $j > 0$  then
10.           $j \leftarrow F[j - 1]$ 
11.      else
12.           $F[i] \leftarrow 0$ 
13.           $i \leftarrow i + 1$ 
```

KMP Algorithm

KMP(T, P), to return the first match

T : String of length n (text), P : String of length m (pattern)

```
1.   $F \leftarrow failureArray(P)$ 
2.   $i \leftarrow 0$ 
3.   $j \leftarrow 0$ 
4.  while  $i < n$  do
5.      if  $T[i] = P[j]$  then
6.          if  $j = m - 1$  then
7.              return  $i - j$  // match
8.          else
9.               $i \leftarrow i + 1$ 
10.              $j \leftarrow j + 1$ 
11.         else
12.             if  $j > 0$  then
13.                  $j \leftarrow F[j - 1]$ 
14.             else
15.                  $i \leftarrow i + 1$ 
16.         return  $-1$  // no match
```

KMP: Example

$P = \text{abacaba}$

j	0	1	2	3	4	5	6
$F[j]$	0	0	1	0	1	2	3

$T = \underline{\text{abaxyabacabbaababacaba}}$

0	1	2	3	4	5	6	7	8	9	10	11
a	b	a	x	y	a	b	a	c	a	b	b
a	b	a	c								
		(a)	b								
			a								
				a							
					a	b	a	c	a	b	a
									(a)	(b)	a

Exercise: continue with $T = \text{abaxyabacabbaababacaba}$

KMP: Analysis

failureArray

- At each iteration of the while loop, at least one of the following happens:
 - 1 i increases by one, or
 - 2 the index $i - j$ increases by at least one ($F[j - 1] < j$)
- There are no more than $2m$ iterations of the while loop
- Running time: $\Theta(m)$

KMP: Analysis

failureArray

- At each iteration of the while loop, at least one of the following happens:
 - 1 i increases by one, or
 - 2 the index $i - j$ increases by at least one ($F[j - 1] < j$)
- There are no more than $2m$ iterations of the while loop
- Running time: $\Theta(m)$

KMP

- failureArray can be computed in $\Theta(m)$ time
- At each iteration of the while loop, at least one of the following happens:
 - 1 i increases by one, or
 - 2 the index $i - j$ increases by at least one ($F[j - 1] < j$)
- There are no more than $2n$ iterations of the while loop
- Running time: $\Theta(n)$

Boyer-Moore Algorithm

Based on three key ideas:

- **Reverse-order searching:** Compare P with a subsequence of T moving backwards
- **Bad character jumps:** When a mismatch occurs at $T[i] = c$
 - ▶ If P contains c , we can shift P to align the last occurrence of c in P with $T[i]$
 - ▶ Otherwise, we can shift P to align $P[0]$ with $T[i + 1]$
- **Good suffix jumps:** If we have already matched a suffix of P , then get a mismatch, we can shift P forward to align with the previous occurrence of that suffix (with a mismatch from the suffix we read). If none exists, look for the longest prefix of P that is a suffix of what we read. **Similar to failure array in KMP.**
- Can skip large parts of T

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								

P = m o o r e

T = b o y e r m o o r e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				

P = m o o r e

T = b o y e r m o o r e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				
											o

P = m o o r e

T = b o y e r m o o r e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				
										d	o

P = m o o r e

T = b o y e r m o o r e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				
									l	d	o

P = m o o r e

T = b o y e r m o o r e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

P = m o o r e

T = b o y e r m o o r e

Bad character examples

$P =$ a l d o

$T =$ w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

6 comparisons (checks)

$P =$ m o o r e

$T =$ b o y e r m o o r e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

6 comparisons (checks)

P = m o o r e

T = b o y e r m o o r e

				e					

Bad character examples

$P =$ a l d o

$T =$ w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

6 comparisons (checks)

$P =$ m o o r e

$T =$ b o y e r m o o r e

				e					
				(r)	e				

Bad character examples

$P =$ a l d o

$T =$ w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

6 comparisons (checks)

$P =$ m o o r e

$T =$ b o y e r m o o r e

				e					
				(r)	e				
					(m)				e

Bad character examples

P = a l d o

T = w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

6 comparisons (checks)

P = m o o r e

T = b o y e r m o o r e

				e					
				(r)	e				
					(m)			r	e

Bad character examples

$P =$ a l d o

$T =$ w h e r e i s w a l d o

			o								
							o				
								a	l	d	o

6 comparisons (checks)

$P =$ m o o r e

$T =$ b o y e r m o o r e

				e					
				(r)	e				
					(m)	o	o	r	e

7 comparisons (checks)

Good suffix examples

$P = \text{sells_shells}$

s h e i l a _ s e l l s _ s h e l l s

$P = \text{odetofood}$

i l i k e f o o d f r o m m e x i c o

Good suffix examples

$P = \text{sells_shells}$

s h e i l a _ s e l l s _ s h e l l s

							h	e	l	l	s							

$P = \text{odetofood}$

i l i k e f o o d f r o m m e x i c o

Good suffix examples

$P = \text{sells_shells}$

s	h	e	i	l	a	_	s	e	l	l	s	_	s	h	e	l	l	s	
							h	e	l	l	s								
							(e)	(l)	(l)	(s)									s

$P = \text{odetofood}$

i	l	i	k	e	f	o	o	d	f	r	o	m	m	e	x	i	c	o	

Good suffix examples

$P = \text{sells_shells}$

s	h	e	i	l	a	_	s	e	l	l	s	_	s	h	e	l	l	s	
							h	e	l	l	s								
							s	(e)	(l)	(l)	(s)	_	s	h	e	l	l	s	

$P = \text{odetofood}$

i	l	i	k	e	f	o	o	d	f	r	o	m	m	e	x	i	c	o

Good suffix examples

$P = \text{sells_shells}$

s	h	e	i	l	a	_	s	e	l	l	s	_	s	h	e	l	l	s	
							h	e	l	l	s								
							s	(e)	(l)	(l)	(s)	_	s	h	e	l	l	s	

$P = \text{odetofood}$

i	l	i	k	e	f	o	o	d	f	r	o	m	m	e	x	i	c	o
				o	f	o	o	d										

Good suffix examples

$P = \text{sells_shells}$

s	h	e	i	l	a	_	s	e	l	l	s	_	s	h	e	l	l	s	
							h	e	l	l	s								
							s	(e)	(l)	(l)	(s)	_	s	h	e	l	l	s	

$P = \text{odetofood}$

i	l	i	k	e	f	o	o	d	f	r	o	m	m	e	x	i	c	o
				o	f	o	o	d										
				(e)					d									

Good suffix examples

$P = \text{sellshells}$

s	h	e	i	l	a	_	s	e	l	l	s	_	s	h	e	l	l	s	
							h	e	l	l	s								
							s	(e)	(l)	(l)	(s)	_	s	h	e	l	l	s	

$P = \text{odetofood}$

i	l	i	k	e	f	o	o	d	f	r	o	m	m	e	x	i	c	o
				o	f	o	o	d										
							(o)	(d)									d	

- Good suffix moves further than bad character for 2nd guess.
- Bad character moves further than good suffix for 3rd guess.
- This is out of range, so **pattern not found**.

Last-Occurrence Function

- **Preprocess** the pattern P and the alphabet Σ
- Build the **last-occurrence function** L mapping Σ to integers
- $L(c)$ is defined as
 - ▶ the largest index i such that $P[i] = c$ or
 - ▶ -1 if no such index exists
- Example: $\Sigma = \{a, b, c, d\}$, $P = abacab$

c	a	b	c	d
$L(c)$	4	5	3	-1

- The last-occurrence function can be computed in time $O(m + |\Sigma|)$
- In practice, L is stored in a size- $|\Sigma|$ array.

Good Suffix array

- Again, we preprocess P to build a table.
- **Suffix skip array** S of size m : for $0 \leq i < m$, $S[i]$ is the largest index j such that $P[i + 1..m - 1] = P[j + 1..j + m - 1 - i]$ **and** $P[j] \neq P[i]$.
- **Note**: in this calculation, any negative indices are considered to make the given condition true (these correspond to letters that we might not have checked yet).
- Similar to KMP failure array, with an extra condition.
- Computed similarly to KMP failure array in $\Theta(m)$ time.

Good Suffix array

Example: $P = \text{bonobobo}$

i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$								

Good Suffix array

Example: $P = \text{bonobob} \circ$

i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$								6

Good Suffix array

Example: $P = \text{bonobobo}$

i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$							2	6

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i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$						-1	2	6

Good Suffix array

Example: $P = \text{bonobobo}$

i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$					2	-1	2	6

Good Suffix array

Example: $P = \text{bonobobo}$

i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$				-3	2	-1	2	6

Good Suffix array

Example: $P = \text{bonobobo}$

i	0	1	2	3	4	5	6	7
$P[i]$	b	o	n	o	b	o	b	o
$S[i]$	-6	-5	-4	-3	2	-1	2	6

- Computed similarly to KMP failure array in $\Theta(m)$ time.

Boyer-Moore Algorithm

boyer-moore(T,P)

1. $L \leftarrow$ last occurrence array computed from P
2. $S \leftarrow$ good suffix array computed from P
3. $i \leftarrow m - 1, \quad j \leftarrow m - 1$
4. **while** $i < n$ **and** $j \geq 0$ **do**
5. **if** $T[i] = P[j]$ **then**
6. $i \leftarrow i - 1$
7. $j \leftarrow j - 1$
8. **else**
9. $i \leftarrow i + m - 1 - \min(L[T[i]], S[j])$
10. $j \leftarrow m - 1$
11. **if** $j = -1$ **return** $i + 1$
12. **else return** FAIL

Exercise: Prove that $i - j$ always increases on lines 9–10.

Boyer-Moore algorithm conclusion

- Worst-case running time $\in O(n + |\Sigma|)$
- This complexity is difficult to prove.
- Worst-case running time $O(nm)$ if we want to report all occurrences
- On typical **English text** the algorithm probes approximately **25%** of the characters in T
- Faster than KMP in practice on English text.

Rabin-Karp Fingerprint Algorithm

Idea: use hashing

- Compute hash function for each text position
- No explicit hash table: just compare with pattern hash
- If a match of the hash value of the pattern and a text position found, then compares the pattern with the substring by naive approach

Rabin-Karp Fingerprint Algorithm

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- Compute hash function for each text position
- No explicit hash table: just compare with pattern hash
- If a match of the hash value of the pattern and a text position found, then compares the pattern with the substring by naive approach

Example:

Hash "table" size = 97

Search Pattern P : 5 9 2 6 5

Search Text T : 3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4 6

Hash function: $h(x) = x \bmod 97$ and $h(P) = 95$.

Rabin-Karp Fingerprint Algorithm

Idea: use hashing

- Compute hash function for each text position
- No explicit hash table: just compare with pattern hash
- If a match of the hash value of the pattern and a text position found, then compares the pattern with the substring by naive approach

Example:

Hash "table" size = 97

Search Pattern P : 5 9 2 6 5

Search Text T : 3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4 6

Hash function: $h(x) = x \bmod 97$ and $h(P) = 95$.

31415 mod 97 = 84

14159 mod 97 = 94

41592 mod 97 = 76

15926 mod 97 = 18

59265 mod 97 = 95

Rabin-Karp Fingerprint Algorithm

Guaranteeing correctness

- Need full compare on hash match to guard against collisions
 - ▶ $59265 \bmod 97 = 95$
 - ▶ $59362 \bmod 97 = 95$

Running time

- Hash function depends on m characters
- Running time is $\Theta(mn)$ for search miss (how can we fix this?)

Rabin-Karp Fingerprint Algorithm

The initial hashes are called **fingerprints**.

Rabin & Karp discovered a way to update these fingerprints in constant time.

Idea:

To go from the hash of a substring in the text string to the next hash value only requires constant time.

- Use previous hash to compute next hash
- $O(1)$ time per hash, except first one

Rabin-Karp Fingerprint Algorithm

Example:

- Pre-compute: $10000 \bmod 97 = 9$
- Previous hash: $41592 \bmod 97 = 76$
- Next hash: $15926 \bmod 97 = ??$

Rabin-Karp Fingerprint Algorithm

Example:

- Pre-compute: $10000 \bmod 97 = 9$
- Previous hash: $41592 \bmod 97 = 76$
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Observation:

$$\begin{aligned} 15926 \bmod 97 &= (41592 - (4 * 10000)) * 10 + 6 \\ &= (76 - (4 * 9)) * 10 + 6 \\ &= 406 \\ &= 18 \end{aligned}$$

Rabin-Karp Fingerprint Algorithm

- Choose table size at **random** to be huge prime
- Expected running time is $O(m + n)$
- $\Theta(mn)$ worst-case, but this is (unbelievably) unlikely

Main advantage:

- Extends to 2d patterns and other generalizations

Suffix Tries and Suffix Trees

- What if we want to search for **many patterns** P within the same **fixed text** T ?
- **Idea:** Preprocess the text T rather than the pattern P
- **Observation:** P is a substring of T if and only if P is a prefix of some suffix of T .

We will call a trie that stores all suffixes of a text T a **suffix trie**, and the compressed suffix trie of T a **suffix tree**.

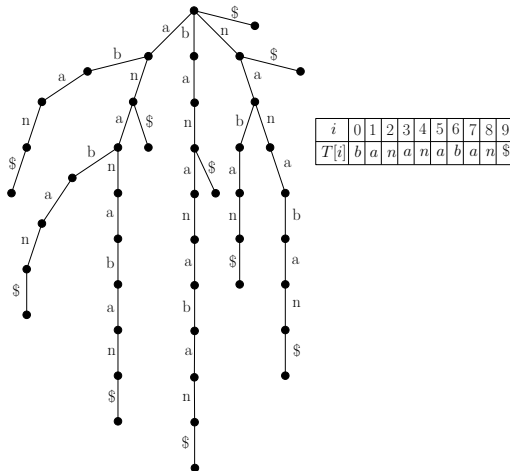
Suffix Trees

- Build the suffix trie, i.e. the trie containing all the suffixes of the text
- Build the suffix tree by compressing the trie above (like in Patricia trees)
- Store two indexes l, r on each node v (both internal nodes and leaves) where node v corresponds to substring $T[l..r]$

Suffix Trie: Example

$T = \text{bananaban}$

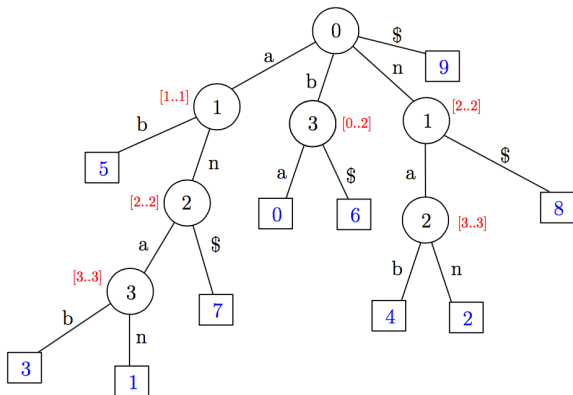
{bananaban, ananaban, nanaban, anaban, naban, aban, ban, an, n}



Suffix Tree (compressed suffix trie): Example

$T = \text{bananaban}$

{ bananaban , ananaban , nanaban , anaban , naban , aban , ban , an , n }



i	0	1	2	3	4	5	6	7	8	9
$T[i]$	b	a	n	a	n	a	b	a	n	\$

Suffix Trees: Pattern Matching

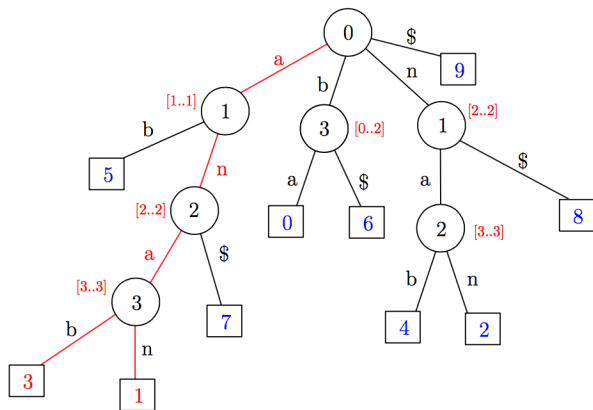
To search for pattern P of length m :

- Similar to Search in compressed trie with the difference that we are looking for a prefix match rather than a complete match
- If we reach a leaf with a corresponding string length less than m , then search is unsuccessful
- Otherwise, we reach a node v (leaf or internal) with a corresponding string length of at least m
- It only suffices to check the first m characters against the substring of the text between indices of the node, to see if there indeed is a match
- We can then visit all children of the node to report all matches

Suffix Tree: Example

$T = \text{bananaban}$

$P = \text{ana}$

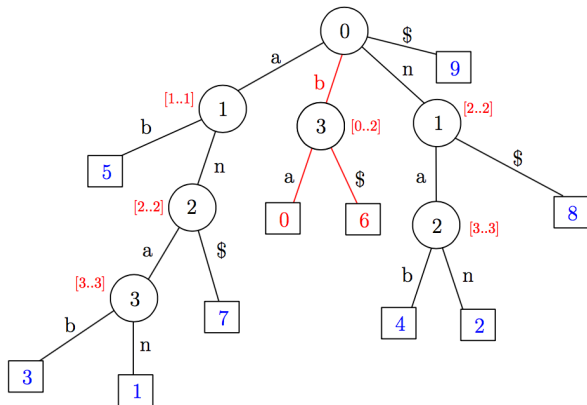


i	0	1	2	3	4	5	6	7	8	9
$T[i]$	b	a	n	a	n	a	b	a	n	\$

Suffix Tree: Example

$T = \text{bananaban}$

$P = \text{ban}$

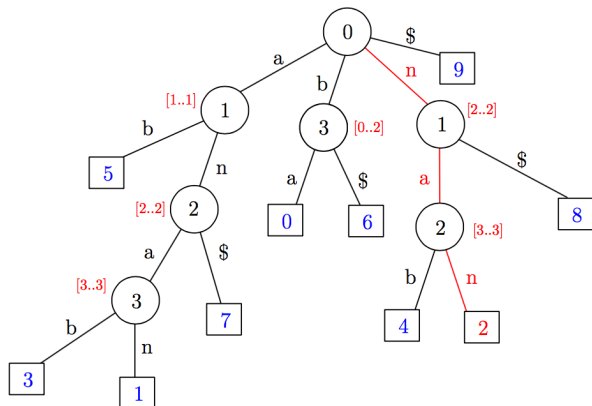


i	0	1	2	3	4	5	6	7	8	9
$T[i]$	b	a	n	a	n	a	b	a	n	\$

Suffix Tree: Example

$T = \text{bananaban}$

$P = \text{nana}$

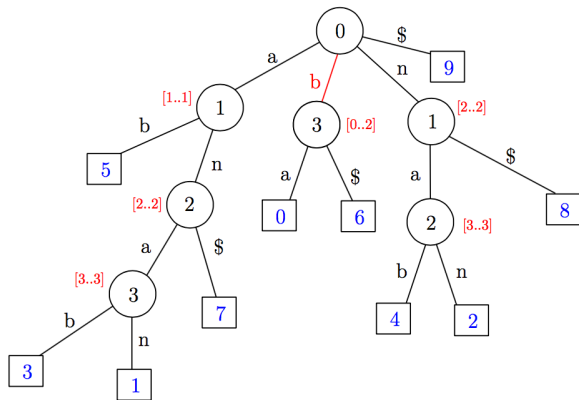


i	0	1	2	3	4	5	6	7	8	9
$T[i]$	b	a	n	a	n	a	b	a	n	\$

Suffix Tree: Example

$T = \text{bananaban}$

$P = \text{bbn}$ not found



i	0	1	2	3	4	5	6	7	8	9
$T[i]$	<i>b</i>	<i>a</i>	<i>n</i>	<i>a</i>	<i>n</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>n</i>	\$
	<i>b</i>	<i>b</i>	<i>n</i>							

Pattern Matching Conclusion

	Brute-Force	DFA	KMP	BM	RK	Suffix trees
Preproc.:	–	$O(m \Sigma)$	$O(m)$	$O(m + \Sigma)$	$O(m)$	$O(n^2)$ ($\rightarrow O(n)$)
Search time:	$O(nm)$	$O(n)$	$O(n)$	$O(n)$ (often better)	$\tilde{O}(n + m)$ (expected)	$O(m)$
Extra space:	–	$O(m \Sigma)$	$O(m)$	$O(m + \Sigma)$	$O(1)$	$O(n)$