

Queen's University
School of Computing

CISC 203: Discrete Mathematics for Computing II
Assignment 3
Due March 21, 2019 at 11:30am

- [5 marks] 1. Suppose we are setting up ten workstations in a computer lab. We want to connect every workstation to a wired network in such a way that all of the networking ports on every workstation are used. Out of the ten workstations, one workstation has 6 networking ports, three workstations have 5 networking ports, four workstations have 4 networking ports, and two workstations have 3 networking ports.

Prove or disprove: it is possible to connect every workstation in such a way that all of the networking ports on every workstation are used.

Hint. Consider the workstations to be vertices in a graph, and consider a workstation's networking ports to be undirected edges incident to the vertex corresponding to that workstation.

- [5 marks] 2. An n -bit Gray code is a set of bit strings of length n that uses a special incrementation method. Instead of the usual binary incrementation method, which increments by flipping one or more bits at a time (i.e., $000 \rightarrow 001 \rightarrow 010 \rightarrow 011 \rightarrow 100 \rightarrow \dots$), a Gray code increments by flipping exactly one bit at a time. As an example, the following sequence lists the elements of a 2-bit Gray code:

$$00 \rightarrow 01 \rightarrow 11 \rightarrow 10.$$

- (a) Illustrate a 3-bit Gray code. Begin by drawing a graph where each vertex corresponds to a bit string of length 3 and where an edge $\{u, v\}$ exists when you can obtain the bit string at vertex v by flipping one bit in the bit string at vertex u . Next, find a Hamiltonian circuit through your graph that corresponds to the sequence of single bit-flips needed to generate your Gray code.
- (b) Name an application where we might prefer to use an n -bit Gray code over the usual binary numbers. Why would we prefer Gray codes in that case?

- [5 marks] 3. Suppose you host a study session and you invite six classmates. Given any pair of people not including yourself, those people may be friends (meaning they know each other) or strangers (meaning they do not know each other).

We can illustrate the property of people being friends or strangers by constructing a graph edge colouring. Given a graph with six vertices, we colour an edge between two vertices red if the people represented by those vertices are friends, or blue if the people are strangers.

Using the notion of graph edge colouring, prove that either at least three people at your study session are pairwise friends or at least three people at your study session are pairwise strangers.

(You should give a general proof. Do not simply draw an example of a graph edge colouring.)

- [5 marks] 4. You have been selected to judge the annual discrete mathematics tournament, where students compete against one another to answer challenging questions and win big prizes.

The rules of the tournament state that all students compete against all other students and, for each match between two students, exactly one of the students wins the match.

With your knowledge of graph theory, you decide to record the outcomes of the tournament in the following way: you create a graph where each student is a vertex and where, given vertices u and v , exactly one directed edge $\{u, v\}$ or $\{v, u\}$ exists in the graph.

If n students compete in the tournament, how many possible "outcome graphs" exist?

Hint. To determine this answer, you must count both the number of ways to choose pairs of students from the set of n students and the outcome of each individual match.