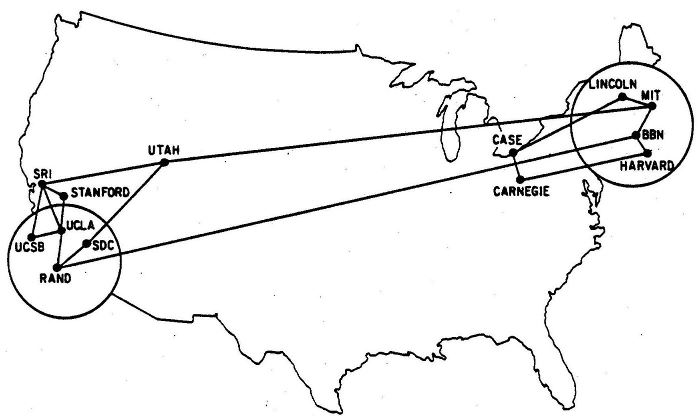
Queen's University
School of Computing

CISC 203: Discrete Mathematics for Computing II
Assignment 4
Due April 5, 2019 at 1:30pm

[5 marks] 1. The Advanced Research Projects Agency Network, or ARPANET, was the predecessor to the internet. The following map depicts ARPANET as it was in December 1970, one year after its creation.



(a) Consider the graph G corresponding to the ARPANET map, where vertices are locations and edges are network connections between locations. (Disregard the circles on the east and west coasts.)

Is G planar? If so, give a planar representation of G. If not, explain why.

(b) Consider again the graph G from part (a). Find $\chi(G)$.

(c) Consider again the graph G from parts (a) and (b). Does G contain an Eulerian path, a Hamiltonian path, or both? Explain why or why not.

[5 marks] 2. In this question, we will consider a particular type of tree structure called a ternary tree. A ternary tree is a tree where every internal vertex has no more than 3 children.

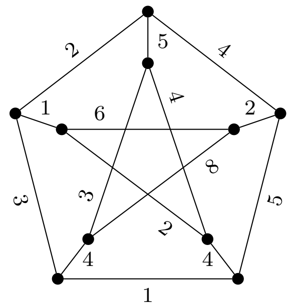
(a) Prove that a ternary tree T of height h contains at most $(3^{h+1} - 1)/2$ vertices.

(b) Suppose we want to store a ternary tree T in an array A. The root of T is stored at position $A[0]$ in the array. Explain how we can store the remaining vertices of T in the array A.

The ternary tree T need not be full, but you may convert T to a full tree by representing "missing" children by some null vertex $\emptyset$.

(c) Given an arbitrary vertex u of T stored at position $A[i]$ in the array from part (b), explain how we can find both the parent of u and all children of u using only direct access to array elements.

[5 marks] 3. Consider the following weighted Petersen graph:



(a) Find a spanning tree for the given weighted Petersen graph. Convert your spanning tree to a rooted tree. What is the height of your rooted spanning tree?

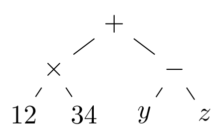
(b) Find a minimum spanning tree for the given weighted Petersen graph. What is the weight of your minimum spanning tree?

(c) Given a connected weighted undirected graph G, a maximum spanning tree is a spanning tree for G that has the largest possible weight.

Briefly describe a method to find a maximum spanning tree for a graph G. Then, use your method to find a maximum spanning tree for the given weighted Petersen graph. What is the weight of your maximum spanning tree?

[5 marks] 4. Some programming languages and compilers represent mathematical expressions in terms of expression trees. In an expression tree, constants/variables are represented by leaves, operations are represented by internal vertices, and the children of an internal vertex are the operands corresponding to the operation at that vertex.

As an example, the expression $(12 \times 34) + (y - z)$ is represented by the following expression tree:



(a) Draw the expression tree for the mathematical expression $(((k + 7) - 1) \times (n + (4 \times m))) - 2$.

(b) If we perform a preorder traversal of an expression tree, we get an expression that is said to be in Polish notation. Similarly, a postorder traversal produces an expression in reverse Polish notation.

Draw the expression tree for the mathematical expression $x - ((yz + x) \times z)$. Write this expression in both Polish notation and reverse Polish notation.