Queen's University
School of Computing

CISC 203: Discrete Mathematics for Computing II
Final Examination Review
Winter 2019

1 Topics Covered

The following list gives an overview of every topic covered in CISC 203. You should ensure you have a good understanding of each topic. All final examination questions will test some topic on this list, but not all topics will be tested on the final examination.

Although the final examination is a comprehensive examination, there will be a greater emphasis placed on topics covered after the midterm examination.

- **Pre-Midterm**
  - Proof techniques
  - Combinatorics
  - Discrete probability
  - Recurrence relations
- **Graph Theory**
  - Definitions
  - Properties of graphs
  - Properties of vertices/edges
  - Special graphs
  - Representing graphs
  - Paths/circuits
  - Connectedness
  - Eulerian paths/circuits
  - Hamiltonian paths/circuits
  - Isomorphisms
  - Planarity
  - Colourings
- **Trees**
  - Definitions
  - Tree terminology
  - Properties of trees
  - Representing trees
  - Tree traversals
  - Spanning trees
  - Depth/breadth-first search
  - Minimum spanning trees
  - Kruskal's algorithm
  - Prim-Jarník algorithm
- **Boolean and Abstract Algebra**
  - Definitions
  - Boolean algebras
  - Groups
  - Properties of groups
  - Examples of groups

2 Format

The final examination is two hours long. It consists of 6 questions worth a total of 100 marks.

The first question is split into 13 multiple-choice style parts. Multiple-choice questions test both pre- and post-midterm material. Each multiple-choice question will be in one of the following formats:

- "choose from A, B, C, D" question;
- match concepts/formulas to definitions; or
- choose all answers that meet some given property (e.g., "Circle the graphs that contain a circuit").

The second question is about pre-midterm material. It is split into two parts. The first part is a proof question, while the second part covers definitions and concepts.

The third question is about graphs. It is split into four parts.

The fourth question is about trees. It is split into three parts.

The fifth question is about both graphs and trees. It is split into three parts.

The sixth question is on abstract algebra. It is split into three parts. Since we covered abstract algebra at the very end of the course, there will not be any lengthy or difficult questions about this topic. Having an understanding of definitions and examples from the lecture notes will suffice.

3 Tips and Tricks

- Double-check the date, time, and room of the final examination. You will not get extra time to write if you arrive late.
- Use your time wisely. The final examination is structured so that you can allocate approximately one minute per mark while still leaving time to review your answers at the end. Thus, for example, a question worth 10 marks should take about 10 minutes to complete.
- Use the resources you are given. The lecture notes contain everything you need to know. The assignment questions are similar in content to the final examination questions, but the difficulty of the final examination questions will be less than the assignment questions. The course textbook and problem sets serve as great supplementary material.
- Don't leave your questions until the last minute. Seek help before the final examination if you have questions. Attend the review session and office hours or send an email.
- Don't try to memorize formulas. Instead, focus on understanding how formulas are derived and how they are applied.
- Don't panic!

4 Practice Questions

The following practice questions are given as a study aid in addition to the assignments and problem sets. Questions on the final examination may be different from the questions given here.

Special thanks to the students who contributed some of the following practice questions.

Proof Techniques

1. Using any appropriate proof technique, prove that the product of an even integer and an odd integer is even.

   Hint. Recall that even integers are of the form $2k$ for some $k \in \mathbb{N}$, while odd integers are of the form $2k + 1$ for some $k \in \mathbb{N}$.

2. Using any appropriate proof technique, prove that

$$0^2 + 1^2 + 2^2 + \cdots + n^2 = \frac{(2n+1)(n+1)(n)}{6}.$$

Combinatorics

1. A homeowner is looking to repaint their walls. They have five rooms: {Living room, Bedroom, Guest bedroom, Kitchen, Basement}.

   If the individual has seven paint options, {White, Yellow, Green, Blue, Red, Orange, Purple}, and they want at least two rooms to share the same colour, how many different ways can they repaint their walls?

2. In a school of 200 children, 15 students are chosen to be on the school's math team, and of those, 2 students are chosen to be co-captains. In how many ways can this be done?

Discrete Probability

1. Jake is playing a tabletop role-playing game called Dungeons & Dragons. The game often revolves around rolling a twenty-sided die, known as a d20. Each face has a number ranging from one to twenty, with an equal probability of rolling each value.

   At one point, to defeat a fearsome monster, Jake must roll two d20s, and at least one of the dice must have a face value of eighteen or above. What is the probability that Jake defeats the monster?

   Example of inputs/outputs:
   (19,2) → 19 (success!)
   (18,4) → 18 (success!)
   (7,8) → 8 (failure.)
   (12,12) → 12 (failure.)

2. Let $(S, \mathbb{P})$ be a sample space where $S = \{1, 2, 3, \ldots, 10\}$ and $\mathbb{P}[j] = j \,/\, 55$ for $1 \leq j \leq 10$. Let A be the event $A = \{1, 4, 7, 9\}$ and let B be the event $B = \{1, 2, 3, 4, 5\}$.
   (a) What is the probability of A?
   (b) What is the probability of B?
   (c) What is $\mathbb{P}[A \mid B]$?
   (d) What is $\mathbb{P}[B \mid A]$?

Recurrence Relations

1. For each of the following recurrence relations, calculate the first six terms of the sequence (that is, a_0 through a_5).

   (a) $a_n = 2a_{n-1} + 2$, $a_0 = 1$.

   (b) $a_n = a_{n-1} + 3$, $a_0 = 5$.

   (c) $a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 0$, $a_1 = 1$.

2. Solve the recurrence relation $a_n = -6a_{n-1} - 8a_{n-2}$ when $a_0 = 1$ and $a_1 = 3$. Give a formula for a_n.

Graph Theory

1. Does there exist a simple graph with six vertices of these degrees? If so, draw such a graph.

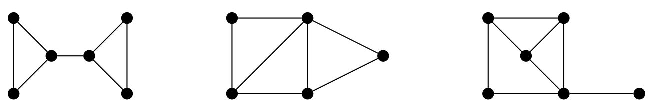
   (a) 0, 1, 2, 3, 4, 5

   (b) 2, 2, 2, 2, 2, 2

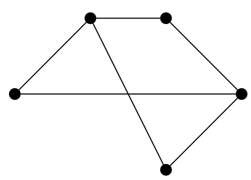
   (c) 3, 2, 2, 2, 2, 3

   (d) 3, 3, 3, 3, 3, 5

2. Determine whether the given graphs have an Eulerian path or a Hamiltonian path.



3. Find the chromatic number of the given graph.



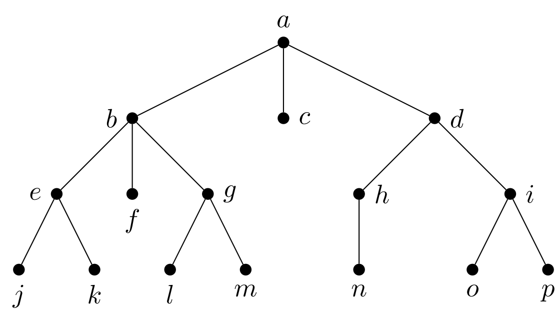
4. A *rook* is a chess piece that may, on a single turn, move any number of squares horizontally or any number of squares vertically on the board. That is, if squares A and B are in the same row (or same column) then we are permitted to move the rook from A to B. But if A and B are in neither the same row nor the same column, a move between these squares is illegal. Thus in every row and every column there are $\binom{8}{2}$ pairs of squares between which the rook may move. This gives a total of $16\binom{8}{2} = 448$ such pairs.

   Suppose a rook is placed on an empty chess board. Can we repeatedly move the rook so that it moves exactly once between each pair of squares in the same row and once between each pair of squares in the same column?

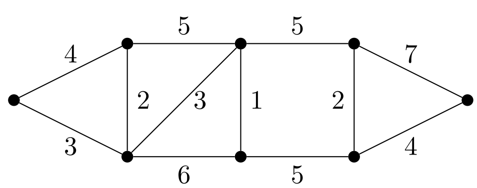
   Note. When the rook travels between squares A and B, it should traverse either from A to B or from B to A, but not both.

Trees

1. Answer these questions about the given rooted tree.
   (a) Which vertex is the root?
   (b) Which vertices are internal vertices?
   (c) Which vertices are leaves?
   (d) Which vertices are children of b?
   (e) Which vertex is the parent of h?
   (f) Which vertices are siblings of g?
   (g) Which vertices are ancestors of m?
   (h) Which vertices are descendants of d?



2. Let G be a connected graph on $n \geq 1$ vertices. Prove that G is a tree if and only if G has exactly $n-1$ edges.
3. Draw the tree T whose pre-order traversal sequence is $[a, b, c, d, e, f, g, h]$ and in-order traversal sequence is $[c, b, d, a, e, g, f, h]$.
4. Find a minimum spanning tree in the given graph using both Kruskal's algorithm and the Prim-Jarník algorithm.



Boolean and Abstract Algebra

1. Define the following terms: binary operation, associativity, distributivity, commutativity, Boolean algebra, identity element, inverse element, group, Abelian group.
2. Consider the operation $x \star y = xy \,/\, (x + y)$, where $x, y \in \mathbb{Z}$.
   (a) Is $(\mathbb{Z}, \star)$ a group?
   (b) Let $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$. Is $(\mathbb{Z}^*, \star)$ a group?