Queen's University
School of Computing

CISC 203: Discrete Mathematics for Computing II
Problem Set 4: Recurrence Relations
Winter 2019

This problem set is provided to you for the purposes of self-study; your solutions will not be marked. Each of these problems, as well as hints and some solutions, can be found in the course textbook.

23.2. Solve each of the following recurrence relations by giving an explicit formula for a_n.

(a) $a_n = \frac{2}{3}a_{n-1}$, $a_0 = 4$.

(b) $a_n = 10a_{n-1}$, $a_0 = 3$.

(c) $a_n = -a_{n-1}$, $a_0 = 5$.

(d) $a_n = 1.2a_{n-1}$, $a_0 = 0$.

(e) $a_n = 3a_{n-1} - 1$, $a_0 = 10$.

(f) $a_n = 4 - 2a_{n-1}$, $a_0 = 0$.

(g) $a_n = a_{n-1} + 3$, $a_0 = 0$.

(h) $a_n = 2a_{n-1} + 2$, $a_0 = 0$.

(i) $a_n = 8a_{n-1} - 15a_{n-2}$, $a_0 = 1$, $a_1 = 4$.

(j) $a_n = a_{n-1} + 6a_{n-2}$, $a_0 = 4$, $a_1 = 4$.

(k) $a_n = 4a_{n-1} - 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$.

(l) $a_n = -6a_{n-1} - 9a_{n-2}$, $a_0 = 3$, $a_1 = 6$.

(m) $a_n = 2a_{n-1} - a_{n-2}$, $a_0 = 5$, $a_1 = 1$.

(n) $a_n = -2a_{n-1} - a_{n-2}$, $a_0 = 5$, $a_1 = 1$.

(o) $a_n = 2a_{n-1} + 2a_n$, $a_0 = 3$, $a_1 = 3$.

(p) $a_n = 2a_{n-1} - 5a_{n-2}$, $a_0 = 2$, $a_1 = 3$.

23.17. There are many types of recurrence relations that are of different forms from those presented in this lecture. Try your hand at finding a formula for a_n for these:

(a) $a_n = na_{n-1}$, $a_0 = 1$.

(b) $a_n = a_{n-1}^2$, $a_0 = 2$.

(c) $a_n = a_0 + a_1 + a_2 + \cdots + a_{n-1}$, $a_0 = 1$.

(d) $a_n = na_0 + (n-1)a_1 + (n-2)a_2 + \cdots + 2a_{n-2} + 1a_{n-1}$, $a_0 = 1$.

23.18. The *Catalan numbers* are a sequence defined by the following recurrence relation:

$$c_0 = 1 \quad \text{and} \quad c_{n+1} = \sum_{k=0}^{n} c_k c_{n-k}.$$

(a) Calculate the first several Catalan numbers, say up to c_8.

(b) Find a formula for c_n.