Queen's University
School of Computing

CISC 203: Discrete Mathematics for Computing II
Problem Set 5: Graph Theory
Winter 2019

This problem set is provided to you for the purposes of self-study; your solutions will not be marked. Each of these problems, as well as hints and some solutions, can be found in the course textbook.

47.2. Draw pictures of the following graphs.

(a) $(\{a, b, c, d, e\}, \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, e\}, \{c, d\}\})$.

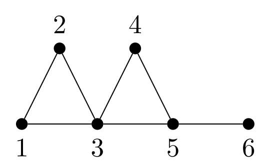
(b) $(\{a, b, c, d, e\}, \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}\})$.

(c) $(\{a, b, c, d, e\}, \{\{a, c\}, \{b, d\}, \{b, e\}\})$.

47.15. Let G be a graph. Prove that there must be an even number of vertices of odd degree.

47.16. Prove that in any graph with two or more vertices, there must be two vertices of the same degree.

48.1. Let G be the graph in the figure.



Draw pictures of the following subgraphs:

(a) $G - 1$.

(b) $G - 3$.

(c) $G - 6$.

(d) $G - \{1, 2\}$.

(e) $G - \{3, 5\}$.

(f) $G - \{5, 6\}$.

49.3. Prove that K_n is connected.

49.4. Let $n \geq 2$ be an integer. Form a graph G_n whose vertices are all the two-element subsets of $\{1, 2, \ldots, n\}$. In this graph we have an edge between distinct vertices $\{a, b\}$ and $\{c, d\}$ exactly when $\{a, b\} \cap \{c, d\} = \emptyset$.

(a) How many vertices does G_n have?

(b) How many edges does G_n have?

(c) For which values of $n \geq 2$ is G_n connected?

49.5. Consider the following (incorrect) restatement of the definition of *connected*: "A graph G is connected provided there is a path that contains every pair of vertices in G."

What is wrong with this sentence?

49.7. How many Hamiltonian paths does a complete graph on $n \geq 2$ vertices have?

51.1. For which values of n is the complete graph K_n Eulerian?

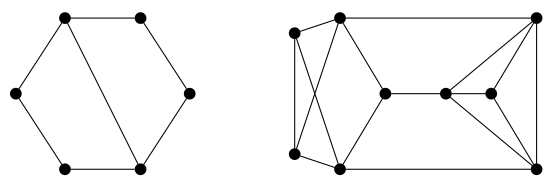
51.2. We noticed that a graph with more than two vertices of odd degree cannot have an Eulerian path, but connected graphs with zero or two vertices of odd degree do have Eulerian paths. The missing case is connected graphs with exactly one vertex of odd degree. What can you say about those graphs?

51.7. A *rook* is a chess piece that may, on a single turn, move any number of squares horizontally or any number of squares vertically on the board. That is, if squares A and B are in the same row (or same column) then we are permitted to move the rook from A to B. But if A and B are in neither the same row nor the same column, a move between these squares is illegal. Thus in every row and every column there are $\binom{8}{2}$ pairs of squares between which the rook may move. This gives a total of $16\binom{8}{2} = 448$ such pairs.

Suppose a rook is placed on an empty chess board. Can we repeatedly move the rook so that it moves exactly once between each pair of squares in the same row and once between each pair of squares in the same column?

Note. When the rook travels between squares A and B, it should traverse either from A to B or from B to A, but not both.

52.1. Let G and H be the graphs in the following figure.



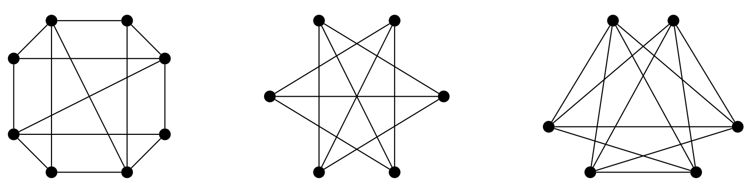
Please find $\chi(G)$ and $\chi(H)$.

52.7. Let G be a graph with n vertices that is not a complete graph. Prove that $\chi(G) < n$.

52.9. Let $G = K_{n,m}$. Determine $|V|$ and $|E|$.

52.17. Suppose G is a graph with 100 vertices. One way to determine whether G is three-colourable is to examine all possible three-colourings of G. If a computer can check 1 million colourings per second, about how long would it take to check all possible three-colourings?

53.2. Each of the graphs in the figure is planar. Redraw these graphs without crossings.



53.6. Let G be a 5-regular graph with ten vertices. Prove that G is nonplanar.