**Queen's University**
**School of Computing**

# CISC 203: Discrete Mathematics for Computing II
# Problem Set 7: Boolean and Abstract Algebra
# Winter 2019

This problem set is provided to you for the purposes of self-study; your solutions will not be marked. Each of these problems, as well as hints and some solutions, can be found in the course textbook.

7.3. Prove: $(x \wedge y) \vee (x \wedge \neg y)$ is logically equivalent to $x$.

7.16. Here is another Boolean operation called *exclusive or*; it is denoted by the symbol $\veebar$. It is defined in the following table.

| $x$ | $y$ | $x \veebar y$ |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

Please do the following:

(a) Prove that $\veebar$ obeys the commutative and associative properties: that is, prove the logical equivalences $x \veebar y = y \veebar x$ and $(x \veebar y) \veebar z = x \veebar (y \veebar z)$.

(b) Prove that $x \veebar y$ is logically equivalent to $(x \wedge \neg y) \vee (\neg x \wedge y)$. (Thus $\veebar$ can be expressed in terms of the basic operations $\wedge$, $\vee$, and $\neg$.)

(c) Prove that $x \veebar y$ is logically equivalent to $(x \vee y) \wedge (\neg(x \wedge y))$. (This is another way that $\veebar$ can be expressed in terms of $\wedge$, $\vee$, and $\neg$.)

(d) Explain why the operation $\veebar$ is called *exclusive or*.

7.19. Prove that $x \vee y$ can be reexpressed in terms of just $\wedge$ and $\neg$.

7.20. Here is yet another Boolean operation called *nand*; it is denoted by the symbol $\overline{\wedge}$. We define $x \overline{\wedge} y$ to be $\neg(x \wedge y)$.

Please do the following:

(a) Construct a truth table for $\overline{\wedge}$.

(b) Is the operation $\overline{\wedge}$ commutative? Associative?

(c) Show how the operations $x \wedge y$ and $\neg x$ can be reexpressed just in terms of $\overline{\wedge}$.

40.1. Let $\star$ be an operation defined on the integers $\mathbb{Z}$ by $x \star y = |x - y|$.

(a) Is $\star$ closed on the integers?

(b) Is $\star$ commutative?

(c) Is $\star$ associative?

(d) Does $\star$ have an identity element? If so, does every integer have an inverse?

(e) Is $(\mathbb{Z}, \star)$ a group?

40.2. Let $\star$ be an operation defined on the real numbers $\mathbb{R}$ by $x \star y = x + y - xy$.

(a) Is $\star$ closed on the real numbers?

(b) Is $\star$ commutative?

(c) Is $\star$ associative?

(d) Does $\star$ have an identity element? If so, does every real number have an inverse?

(e) Is $(\mathbb{R}, \star)$ a group?

40.9. Prove that if $(G, *)$ is a group and $g \in G$, then $\left(g^{-1}\right)^{-1} = g$.

40.10. Prove that if $(G, *)$ is a group, then $e^{-1} = e$.

40.17. Let $(G, *)$ be a group and let $g, h \in G$. Prove that $(g * h)^{-1} = h^{-1} * g^{-1}$.