St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 2
Due March 10, 2022 at 1:15pm

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[6 marks] 1. You have been hired by a large blue-and-yellow furniture store in Halifax to work in their logistics department. Every September, this store receives a large amount of orders for furniture to be shipped to Antigonish, and they must send some number of trucks each day between Halifax and Antigonish. All trucks have a fixed weight limit W, and each furniture item i has an associated fixed weight w_i. The store's loading dock is small, so only one truck can be loaded at a time, and company policy dictates that items be shipped in the same order as they were purchased.

The store's strategy for shipping is to use a greedy approach: pack furniture onto the truck in the order it was purchased, and whenever the next item of furniture would put the truck over its weight limit, save that item for the next truck and dispatch the current truck. However, the company isn't sure whether this is the best strategy, so they want you to verify this.

Prove that, for a given set of furniture with associated weights, their greedy approach indeed minimizes the number of trucks that are used. Your analysis should use the "greedy stays ahead" strategy to establish optimality, as we saw in lecture.

[8 marks] 2. The Winter Olympics are introducing a new event called "winter triathlon", where competitors must swim 1.5km in a frigid lake, steer a dogsled for 40km, and finish with a 10km cross-country ski. Due to a shortage of lifeguards, however, only one competitor is allowed to be in the lake at a time. Thus, the organizers have decided to run the event as follows: the first competitor swims in the lake, and once they finish and get out of the lake, the second competitor begins to swim. Multiple competitors can dogsled or ski at once.

Each competitor i has a projected swim time s_i, a projected dogsled time d_i, and a projected ski time c_i. The organizers must use this information to create a schedule of the competitors' start times, and since it's cold outside, they want the completion time (i.e., the time when all competitors have finished the event) to be as early as possible.

(a) What is the optimal strategy for sequencing competitors in order to minimize the completion time? Briefly explain your strategy. Your answer should be in terms of any of the values s_i, d_i, and c_i, but keep in mind that only one competitor at a time is allowed to swim.

(b) Show that your strategy is, in fact, optimal. Your analysis should use the "exchange argument" strategy to establish optimality, as we saw in lecture.

[6 marks] 3. Given a graph $G = (V, E)$, let us call G an *almost-tree* if G is connected and G contains at most $n + 12$ edges, where $n = |V|$. Each edge of G has an associated cost, and we may assume that all edge costs are distinct.

Describe an algorithm that takes as input an almost-tree G and returns a minimum spanning tree of G. Your algorithm should run in $O(n)$ time.

[5 marks] 4. Suppose you are given a connected graph G, where each edge of G has an associated cost. Further suppose that each edge cost is distinct. We know that it is possible to find a minimum spanning tree of G, but we can go one step further.

Prove that G has a *unique* minimum spanning tree.