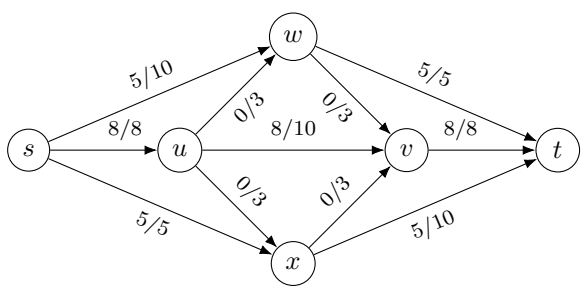
St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 4
Due April 14, 2022 at 1:15pm

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[12 marks] 1. Consider the following flow network.



(a) What is the value of the flow shown in the network? Is this a max flow? If it is, explain why. If it is not, find a max flow.

(b) Find a min cut in the flow network, and give the capacity of your min cut.

[8 marks] 2. The university has decided to invest everyone's tuition money into developing a student-only high-speed wireless network. During a trial period, the university tests their new network with n students, and each student is represented by an (x, y) coordinate. Each student will connect to one of several base stations located on campus, which are also represented by (x, y) coordinates.

Each student connects to exactly one base station, with the connection being constrained by two factors: a range parameter r, meaning that the student must be within distance r of the base station, and a load parameter ℓ, meaning that the base station cannot handle more than ℓ students.

Given the coordinates of the n students, the coordinates of the base stations, the range parameters of each student, and the load parameters of each base station, design an algorithm to decide whether all n students can be simultaneously connected to some base station (not necessarily the same one), subject to the given range and load conditions.

Explain briefly how your algorithm decides this problem. You do not need to prove the runtime of your algorithm.

[5 marks] 3. A hot new startup, CompuGlobalHyperMegaNet, is developing a high-performance distributed computer where asynchronous processes can use a shared set of processing resources. The computer has m available resources and n processes to run, and each process requests a subset of resources that it wants to use. Each resource can be requested by multiple processes, but it can only be used by one process at a time.

The startup's major problem is to figure out how best to allocate resources to the processes that request them. If a process is allocated all of the resources it requested, we say it is *active*. The optimal allocation will result in as many processes as possible being active.

This problem can be summarized as the *Resource Reservation Problem*: given an integer k and sets of n processes and m resources, with each process requesting a subset of resources, is it possible to allocate resources to processes such that at least k processes are active?

(a) First, consider the case where $k = 2$. Explain how we can solve this special case of the Resource Reservation Problem in polynomial time.

(b) Now, consider the general case. Can we solve the Resource Reservation Problem in polynomial time, or is the problem NP-complete? (If the problem is NP-complete, show this via a reduction from another NP-complete problem.)