

St. Francis Xavier University
Department of Computer Science
CSCI 544: Computational Logic
Assignment 1
Due February 11, 2022 at 11:15am

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
 - Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
 - You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.
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- [4 marks] 1. For each of the following formulas, construct a truth table *and* depict the tree representation of the formula. Show all intermediate steps in your truth table.
- (a) $A = \neg(p \Leftrightarrow \neg(q \wedge \neg p))$.
- (b) $B = (p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$.
- [4 marks] 2. For each of the following substitutions, prove that the two formulas are indeed logically equivalent.
- (a) $A \wedge B \equiv \neg(\neg A \vee \neg B)$.
- (b) $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$.
- [4 marks] 3. Given a set C of logical connectives, we say that the set is *adequate* if all other logical connectives can be expressed as some combination of only the connectives in C .
- (a) Show that each of the connectives in the set $\{\wedge, \Rightarrow, \Leftrightarrow\}$ can be expressed in terms of the other two connectives.
- (b) Explain why the set $\{\wedge, \vee\}$ is *not* an adequate set of connectives.
Hint. Suppose we have an interpretation assigning true to every formula. What will the truth value be of a formula using only \wedge and \vee , and what does this imply?
- [6 marks] 4. Using the method of semantic tableaux, determine whether each of the following formulas are valid. If a formula is not valid, give an example of an interpretation demonstrating this. (Remember that a formula A is valid if and only if $\neg A$ is unsatisfiable.)
- (a) $A = (p \wedge q) \Rightarrow (p \vee r)$.
- (b) $B = (p \vee \neg(q \wedge r)) \Rightarrow ((p \Leftrightarrow r) \vee q)$.
- [6 marks] 5. Prove the validity of each of the following sequents using natural deduction.
- (a) $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$.
- (b) $q \Rightarrow r \vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.
- [6 marks] 6. Using natural deduction, prove the following:

$$((p \Rightarrow q) \Rightarrow q) \Rightarrow ((q \Rightarrow p) \Rightarrow p).$$