St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 2
Due February 16, 2023 at 1:15pm

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[5 marks] 1. Suppose we are given a connected graph $G = (V, E)$ and a specific vertex $u \in V$. Using our knowledge of graph search algorithms from our lectures, we construct a depth-first search tree rooted at u that contains all vertices of G, and we call this tree T. We then construct a breadth-first search tree again rooted at u, and we happen to obtain the same tree T.

Prove that $G = T$. (In other words, if T is both a depth-first search tree and a breadth-first search tree rooted at u, then G cannot contain any edges that do not also belong to T.)

[6 marks] 2. You have been hired by the Nova Scotia Department of Public Works as a technical consultant for a new highway under construction. This highway is in a rural part of the province, so communications are limited. The government wants to place emergency phones at certain points along the length of the highway so that motorists can contact somebody if they need help.

Your goal as consultant is to figure out how many phone towers need to be constructed so that every emergency phone is within four kilometres of one of the towers. Give an algorithm that finds a solution for this problem using as few towers as possible, and prove that your algorithm produces an optimal solution.

[7 marks] 3. Before every exam period, the university faces a scheduling problem of a certain type. Professors must have their exam papers printed, and the university has a limited number of photocopiers. Each morning, the university receives a set of print jobs from professors, with each job i requiring time t_i to complete. When a print job i is scheduled, it also has a completion time C_i. Lastly, each professor has a weight w_i that indicates how important their exam is.

The university decides to order print jobs in such a way that the weighted sum of the completion times, $\sum_{i=1}^{n} w_i C_i$, is minimized. Design an algorithm that takes a set of n print jobs, each with a processing time t_i and weight w_i, and produces a schedule that minimizes this weighted sum of completion times. Prove that your algorithm produces an optimal solution.

[7 marks] 4. StFX has received a tip that a malicious student has acquired equipment to clone student cards. The university has confiscated n cards that they suspect are fraudulent. Every card has a magnetic stripe that corresponds to a student account, and if two magnetic stripes correspond to the same student account, we say the cards are *equivalent*.

Instead of using the data printed on the front of the card (which could be forged), the university has a magnetic stripe reader that takes two cards and determines whether the cards are equivalent.

You must determine, among the university's set of n cards, whether there is a subset of more than $n/2$ cards that are all equivalent to one another. You may only take two actions with the set of cards: select two cards at a time, and insert two cards into the reader. Devise a procedure to solve this problem that only requires you to use the card reader $O(n \log(n))$ times, and justify the time complexity of your procedure.