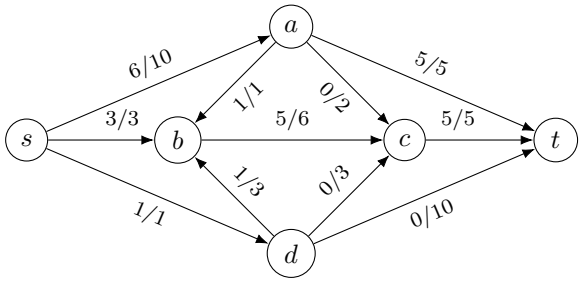
St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 4
Due April 3, 2023 at 1:15pm

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[7 marks] 1. Consider the following flow network.



(a) What is the value of the flow shown in the network? Is this a max flow? If it is, explain why. If it is not, find a max flow.

(b) Find a min cut in the flow network, and give the capacity of your min cut.

[5 marks] 2. Following a natural disaster, paramedics must help a total of n injured people located across the county by transporting them to one of k hospitals in the county. Each of the n people must be dropped off at a hospital that is within a 30 minute drive of their current location. However, the paramedics cannot put any single hospital over capacity by transporting too many patients there. Each hospital can receive at most $\lceil n/k \rceil$ people.

Give a polynomial-time algorithm that takes a list of injured people, a list of hospitals, and information about each person's current location, and determines whether it is possible for the paramedics to transport all people without placing any hospital over capacity.

[8 marks] 3. The department is asking you to organize a student seminar next year that will meet once per week in the fall term. The plan for this seminar is to have the first half of the term consist of a series of ℓ guest lectures by researchers, and to have the second half of the term consist of a set of p applied projects that students will work on.

A total of n researchers volunteered to give guest lectures, and for each week i, a subset L_i of these researchers will be available to speak. On the other hand, each applied project j requires students to have attended at least one lecture out of a subset of P_j guest lectures in order to complete the project.

We can formalize this as the *lecture scheduling problem*: given a set of n researchers and a set of p applied projects, can you select exactly one speaker for each of the first ℓ weeks of the term such that (i) each speaker is available in their scheduled week, and (ii) for each project j, students will have attended at least one of the lectures in the subset P_j?

(a) Suppose that $\ell = 2$, $n = 4$, and $p = 3$. Our speakers are labelled A, B, C, and D, and our subsets are as follows:

$$\begin{aligned} L_1 &= \{A, B, C\} & P_1 &= \{B, C\} \\ L_2 &= \{A, D\} & P_2 &= \{A, B, D\} \\ & & P_3 &= \{C, D\} \end{aligned}$$

Is this a positive ("yes") instance of the lecture scheduling problem? Explain why or why not.

(b) Prove that the lecture scheduling problem is in NP.

(c) Prove that the lecture scheduling problem is NP-complete.
Hint. Try reducing from 3-SATISFIABILITY.

[5 marks] 4. Choose your favourite topic from the course, and write a multiple-choice style question with one correct answer and 3–4 plausible-but-incorrect answers that tests a concept or notion related to that topic.

For inspiration, consider the multiple-choice style questions you saw on the midterm and practice midterm exams.