St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Final Examination
April 22, 2022
9:00am–11:30am

Student Name: ______________________________

Email Address: ______________________________

Instructor: T. J. Smith (Section 20)

Format:

The exam is 150 minutes long. The exam consists of 6 questions worth a total of 70 marks. The exam booklet contains 9 pages, including the cover page and one blank page at the back of the exam booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your exam booklet contains 9 pages. Do not detach any pages from your exam booklet.
4. Do not use any unauthorized reference materials or devices during this exam.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: ______________________________

Question	Marks	Score
1	10	
2	10	
3	14	
4	14	
5	12	
6	10	
Total	70	

Multiple Choice

[10 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

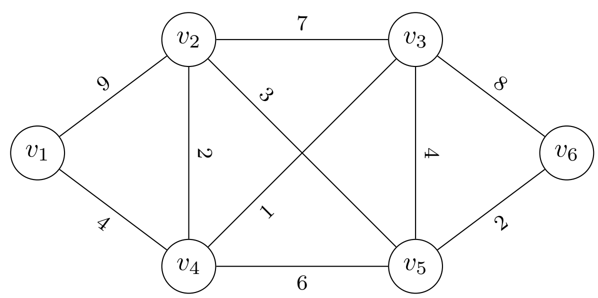
(a) Which of the following functions grows fastest?

A. $f_1(n) = 2^{\log(n)}$.
B. $f_2(n) = n \cdot 2^n$.
C. $f_3(n) = e^n$.
D. $f_4(n) = \sqrt{n}$.

(b) Which of the following is **not** an algorithm to compute a minimum spanning tree in a graph?

A. Borůvka's algorithm.
B. Dijkstra's algorithm.
C. Kruskal's algorithm.
D. Prim's algorithm.

(c) Consider the following graph. What is the length of the shortest path from vertex v_1 to vertex v_6?



A. 15.
B. 12.
C. 11.
D. 9.

(d) Which of the following statements is **false**?

A. Mergesort can run in linear time using median-of-medians selection.
B. Both mergesort and quicksort are comparison-based sorting algorithms.
C. Quicksort partitions its input array into three subarrays to sort elements.
D. Mergesort is a deterministic algorithm, while quicksort is a randomized algorithm.

(e) Consider the following recurrence relation:

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ 2T(n/2) + n & \text{if } n > 1. \end{cases}$$

Using the master theorem, what can you conclude about the growth rate of $T(n)$?

A. $T(n) \in \Theta(n \log(n))$.
B. $T(n) \in \Theta(n)$.
C. $T(n) \in \Theta(2T(n/2))$.
D. $T(n) \in \Theta(2^n)$.

(f) Which of the following is a good indicator that a problem can be solved using dynamic programming?

A. The problem is composed of smaller independent subproblems.
B. The problem is composed of smaller overlapping subproblems.
C. The problem requires that we make irreversible decisions as we compute the solution.
D. We tried everything else and we ran out of ideas.

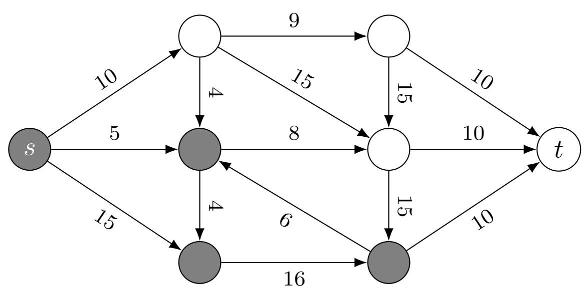
(g) Consider the following instance of the knapsack problem: you are given a knapsack with a capacity of 11 kg, and you can choose from the following items:

i	v_i	w_i
1	\$1	1 kg
2	\$6	2 kg
3	\$18	5 kg
4	\$22	6 kg
5	\$28	7 kg

Assuming you add items according to maximum value, which items will you add to your knapsack?

A. $\{1, 2, 5\}$.
B. $\{3, 4\}$.
C. $\{1, 2, 3\}$.
D. $\{5\}$.

(h) What is the capacity of the cut (depicted by the gray vertices) in the following graph?



A. 9.
B. 19.
C. 28.
D. 46.

(i) If we know that Y is a decision problem that can be solved in polynomial time, and we also know that $X \leq_P Y$ for some decision problem X, then what can we conclude about X?

A. X can be solved in polynomial time.
B. X may or may not be able to be solved in polynomial time.
C. X cannot be solved in polynomial time.
D. X is NP-complete.

(j) Which of the following results is **<u>not</u>** known?

A. $\mathsf{P} \subseteq \mathsf{NP}$.
B. $\mathsf{P} \neq \mathsf{EXP}$.
C. $\mathsf{NP} \subseteq \mathsf{EXP}$.
D. $\mathsf{NP} \neq \mathsf{EXP}$.

Short Answer

[10 marks] 2. For each of the following questions, give a 1–2 sentence answer.

(a) What is a difference between Big-O notation and Big-Omega notation?

(b) Why do all deterministic comparison-based sorting algorithms have a lower bound of $\Omega(n \log(n))$ in the worst case? (You do not need to give a formal proof; an intuitive explanation will suffice.)

(c) Briefly explain a difference between top-down and bottom-up dynamic programming.

(d) What does it mean for an algorithm's runtime to be pseudopolynomial?

(e) Describe some considerations you might make when determining which algorithm design technique is most appropriate for a given problem.

[14 marks] 3. (a) Consider the recurrence relation $T(n) = 2T(n/2) + n^4$. For this recurrence relation,

- sketch the recurrence tree for $T(n)$;
- identify the values a, b, and c; and
- give an asymptotic bound on its growth rate using the master theorem.

(b) Consider a complete binary tree with $n = 2^h - 1$ vertices, where h is the height of the tree. Each vertex v of the tree is labelled by a distinct real number x_v. Moreover, for each vertex v, we can only determine the value x_v by probing that vertex directly; we cannot access the values in any other way (e.g., in an array).

We say that a vertex v is a local minimum if the label x_v is less than the labels x_u for all vertices u that are connected to v by a single edge.

Given such a complete binary tree, describe an algorithm to find *some* local minimum in the tree using only $O(\log(n))$ probes. (Note that you do not need to find the absolute minimum value; only some local minimum suffices.)

[14 marks] 4. Some people in the business school are getting worried about the volatility of the stock market, so they've come to you with a simple question: when should we sell our stocks to maximize our profit?

Suppose you look at the price of a stock over n consecutive days, numbered 1 through n. For each day i, the stock has a price of $p(i)$ per share on that day. For simplicity's sake, you can assume the price remains the same for the entire day i.

In order to make a profit, the stock must be bought on day i and sold on day $j > i$ such that $p(j) - p(i) > 0$. Your job is to figure out the maximum possible profit you can realize by selling the stock.

(a) Let R_j denote the profit (or loss) realized if you sell the stock on day j, where $1 \leq j \leq n$. Consider the following table listing the price of one share of stock over the past week:

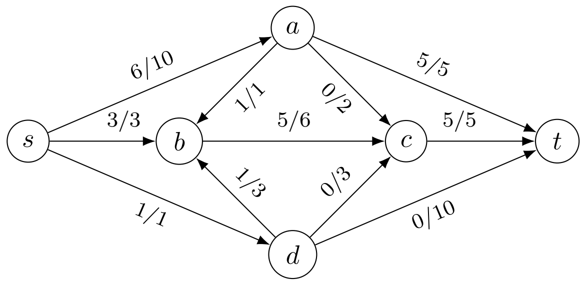
i	1	2	3	4	5	6	7
$p(i)$	20	15	19	23	23	30	28

Compute R_j for all $1 \leq j \leq 7$ in this example, assuming you buy one share of the stock on day 1. Which day was the optimal day for you to sell your share?

(b) Suppose that you can only access a stock's price from yesterday and today, meaning that you can no longer simply compute the difference between today's price and the price on day 1. Using only the values R_{j-1}, $p(j-1)$, and $p(j)$, give a formula to compute R_j.

(c) Clearly, you do not want to sell the stock on days when R_j is negative. In this case, you can take R_j to be 0, since you realize no profit on that day. Using your formula from part (b), give an expression to find the *maximum possible* profit you can realize from selling your stock on some day $1 \leq j \leq n$.

[12 marks] 5. Consider the following flow network.



(a) Draw the residual network corresponding to the given flow network.

(b) What is the value of the flow shown in the network?

(c) Find a min cut in the flow network, and give the capacity of your min cut.

(d) Using your answer from part (c), can you conclude that the flow shown in the network is a max flow? Why or why not?

[10 marks] 6. Recall some of the NP-complete decision problems we defined in class:

A.	Independent set	D.	Knapsack
B.	Vertex cover	E.	3-colour
C.	Set cover	F.	Subset sum

Each of the problem statements below can be formulated as an instance of one of the above decision problems. For each problem statement below, match it to the most appropriate decision problem, and give a brief justification of your decision.

(a) The university is starting a student emergency response team on campus. Volunteers are scheduled in pairs, and some volunteers may be placed in more than one pair. The university wants to run a first aid training session so that, in every pair, at least one volunteer has been trained. The university would also like to keep the size of the training session as small as possible.

(b) A nuclear power plant is assigning jobs to employees. Each job has a priority rating from 1 to 100 and a radiation exposure amount from 1 to 100. The government regulates the amount of radiation exposure each employee can be subjected to in one day. The power plant wishes to assign jobs to each employee in such a way that the total priority of the jobs is maximized and the employee's daily radiation exposure limit is not exceeded.

[2 marks] *Bonus.* What was your favourite part of this course, and why?

This blank page may be used for rough work.