St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Midterm Examination
March 2, 2022
11:15am–12:05pm

Student Name: _____

Email Address: _____

Instructor: T. J. Smith (Section 20)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: _____

Question	Marks	Score
1	5	
2	5	
3	5	
4	10	
Total	25	

Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Consider the following instance of the stable matching problem:

	1st	2nd	3rd
StFX	Hakin	Saini	Ricketts
Dal	Saini	Hakin	Ricketts
Acadia	Hakin	Ricketts	Saini

	1st	2nd	3rd
Hakin	StFX	Dal	Acadia
Saini	Dal	StFX	Acadia
Ricketts	StFX	Acadia	Dal

Which of the following is a stable matching produced by the Gale–Shapley algorithm?

A. { StFX–Saini, Dal–Ricketts, Acadia–Hakin }
B. { StFX–Hakin, Dal–Saini, Acadia–Ricketts }
C. { StFX–Hakin, Dal–Ricketts, Acadia–Saini }
D. { StFX–Saini, Dal–Hakin, Acadia–Ricketts }

(b) What does it mean for a function $f(n)$ to be Big-O of another function $g(n)$?

A. The function $f(n)$ is always larger than the function $g(n)$.
B. For all large-enough values of n, some constant times $f(n)$ is bounded below by $g(n)$.
C. We can replace $g(n)$ with $f(n)$ in any expression to produce an equivalent expression.
D. For all large-enough values of n, $f(n)$ is bounded above by some constant times $g(n)$.

(c) Which of the following functions $f(n)$ is **not** $O(g(n))$?

A. $f(n) = 21n^2 + n(1 + \log(n))$, $g(n) = n^3$
B. $f(n) = 42n^8 + 16n^6 + 5n\log(n)$, $g(n) = n^{10}$
C. $f(n) = n \cdot 2^n$, $g(n) = 2^n$
D. $f(n) = \log(n^2)$, $g(n) = \log(n)$

(d) Which property ensures that the cashier's algorithm returns an optimal solution?

A. The set of coin denominations is exactly $\{1, 5, 10, 25, 100\}$.
B. The set of coin denominations $\{c_1, \ldots, c_n\}$ is ordered $c_1 < \cdots < c_n$ with $c_1 = 1$.
C. The set of coin denominations $\{c_1, \ldots, c_n\}$ is ordered $c_1 < \cdots < c_n$ with $c_1 > 1$.
D. Any arbitrary set of coin denominations is given as input.

(e) Which of the following is **not** an equivalent characterization of a spanning tree H of a graph $G = (V, E)$?

A. H is acyclic and connected.
B. H contains $|V|$ edges.
C. H is maximally acyclic.
D. H is minimally connected.

Short Answer

[5 marks] 2. Consider the following pseudocode. State the runtime of this algorithm in terms of Big-O notation, assuming n is the input size. Also give a brief (2–3 sentence) justification of your answer.

You can assume that assignments and arithmetic operations take $O(1)$ time. Note that you should give the tightest upper bound possible as your answer.

Algorithm: Mystery algorithm

```
i ← n
s ← 0
while i > 0 do
    j ← n
    for 0 ≤ x ≤ n do
        s ← s + x
    while j > 0 do
        s ← s + j
        j ← j/2
    i ← i/2
```

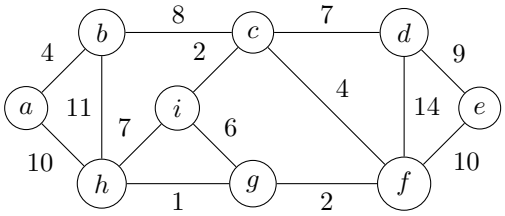
[5 marks] 3. You are a network technician working for the university, and you are installing the wireless network for a brand new building on campus. In this building, there is a long hallway with professors' offices at irregular intervals. The hallway is bounded by a northern stairwell and a southern stairwell.

The professors want to make sure they have consistent access to the internet, so they want their office door to be within 3 metres of a wireless access point. Give a greedy algorithm that achieves this goal using as few wireless access points as possible.

You can either describe your algorithm in plain English, or give pseudocode to describe your algorithm. You do not need to prove the correctness or running time of your algorithm.

[10 marks] 4. (a) Give one similarity and one difference between *Prim's algorithm* and *Kruskal's algorithm*.

(b) Using any appropriate algorithm, find a minimum spanning tree for the following graph. Specify which algorithm you are using. You do not need to draw every step performed by the algorithm; you only need to give the sequence of vertices or edges added by the algorithm. For clarity, draw your final minimum spanning tree on the graph as well.



This blank page may be used for rough work.