St. Francis Xavier University
Department of Computer Science

CSCI 435: Algorithms and Complexity
Assignment 3
Due April 4, 2023 at 8:15am

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[13 marks] 1. You have been hired as a consultant for the Halifax Port Authority. They handle millions of dollars of cargo each year, but their revenue is constrained by the rate at which they can unload the cargo from the ships in port.

Suppose a ship arrives holding n containers having integer weights w_1 through w_n. On land is a set of trucks, each of which is capable of holding some integer K units of weight. Each truck can accommodate multiple containers, as long as the total weight of any one truck does not exceed K. Your job is to minimize the number of trucks needed to transport all containers.

The natural approach is to use a greedy algorithm: starting with an empty truck, workers load containers 1, 2, 3, ..., onto the truck until reaching a container that would exceed the weight limit K. They then send the truck off, and begin loading another empty truck in the same way starting from the current container.

(a) As is often the case, the greedy algorithm is not optimal for this problem. Give an example of a set of weights $\{w_1, \ldots, w_n\}$ and a value K where the greedy algorithm produces a suboptimal answer.

(b) Show that the number of trucks loaded by the greedy algorithm is within a factor of 2 of the optimal answer for any set of weights $\{w_1, \ldots, w_n\}$ and value K, thus giving us a 2-approximation algorithm.

[12 marks] 2. The property of resiliency is crucial when designing and constructing a communications network. (A certain Canadian telecommunications company learned this the hard way in July 2022, when their entire network failed for a day.)

We can model network resiliency in the following way. Think of our network as an undirected graph $G = (V, E)$, where each edge $e \in E$ has an associated *cost* $c_e \geq 0$ and each pair of vertices $u, w \in V$, $u \neq w$, has a non-negative integer *connectivity requirement* r_{uw}. In the *network resiliency problem*, we must find a minimum-cost subset of edges $F \subseteq E$ such that, for all pairs of vertices $u, w \in V$, $u \neq w$, there exist at least r_{uw} edge-disjoint paths connecting u and w in our network.

Create an integer program to model the network resiliency problem, and additionally describe your objective function and each constraint in plain English. Then, explain how you can relax your integer program in an appropriate way to get the corresponding linear program.

Hint. One of your constraints will need to maintain the connectivity requirement by ensuring that a sufficient number of edges remain if we were to cut some subset of vertices $S \subseteq V$ from the remainder of the graph.

[5 marks] 3. Choose your favourite topic from the course, and write a multiple-choice style question with one correct answer and 3–4 plausible-but-incorrect answers that tests a concept or notion related to that topic.

For inspiration, consider the multiple-choice style questions you saw on the midterm exam.