

St. Francis Xavier University
Department of Computer Science
CSCI 544: Computational Logic
Assignment 2
Due March 10, 2023 at 11:15am

[8 marks] 1. Let P , Q , and R be predicates taking two, two, and three arguments, respectively. Consider the formula

$$A = \forall x ((P(x, y) \Rightarrow P(x, x)) \vee (Q(y, z) \wedge \exists y R(x, y, z))).$$

- (a) Draw the tree representation of A .
- (b) Identify all bound variables and free variables in A . Do there exist any variables in A that have both bound and free occurrences?
- (c) Give the resultant formula after performing the substitution $A[y/f(x)]$. Is $f(x)$ free for y in A ?

[6 marks] 2. (a) Prove that the two formulas $\exists x (P(x) \Rightarrow Q(x))$ and $\forall x P(x) \Rightarrow \exists x Q(x)$ are logically equivalent using any of the propositional or predicate logic identities we learned in lecture.

(b) Suppose that we allow the domain of an interpretation to be empty, and consider the equivalence

$$\forall y P(y) \vee \exists x Q(x) \equiv \exists x (\forall y P(y) \vee Q(x)).$$

Does the equivalence hold or not hold under an “empty interpretation”? Explain why.

[8 marks] 3. Using the method of semantic tableaux, determine whether the following formula is valid:

$$\forall x (P(x) \vee Q(x)) \Rightarrow (\forall x P(x) \vee \exists x Q(x)).$$

For ease of writing, you can use the list format to prove validity instead of the tree format.

[8 marks] 4. Prove the validity of each of the following sequents using natural deduction.

- (a) $\exists x (S \Rightarrow Q(x)) \vdash S \Rightarrow \exists x Q(x)$.
- (b) $\exists x (P(x) \wedge Q(x)) \vdash \exists x P(x) \wedge \exists x Q(x)$.