St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 1
Due January 25, 2024 at 1:30pm

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[7 marks] 1. The Smith family cats all have a very vocal preference for which flavour of treats they prefer to eat. At the same time, certain flavours of treats are better for the digestive systems of certain cats. Suppose we have the following two preference lists, where the list on the left is cats' preference for treats ordered from most favourite to least favourite, and the list on the right is treats' "preference" ordered by best to worst for the cat's system:

Treelo	Chicken	Tuna	Beef	Cheddar	**Chicken**	Treelo	Eviolet	Darla	Jesse
Eviolet	Cheddar	Tuna	Chicken	Beef	**Tuna**	Jesse	Eviolet	Treelo	Darla
Darla	Cheddar	Beef	Chicken	Tuna	**Cheddar**	Jesse	Darla	Treelo	Eviolet
Jesse	Tuna	Chicken	Beef	Cheddar	**Beef**	Darla	Treelo	Eviolet	Jesse

Using the Gale–Shapley algorithm, find a stable matching that is cat-optimal. Show all your work.

(If you like, you may implement the Gale–Shapley algorithm in the programming language of your choice, but your answer must include a copy of your source code and a verbose listing of each step of your implementation's output.)

[6 marks] 2. In the original paper introducing the Gale–Shapley algorithm, the authors framed the stable matching problem in terms of men and women "rank[ing] those of the opposite sex in accordance with his or her preferences for a marriage partner". In 1962, when this paper was published, society's views on marriage were more conservative than they are today. So let's bring the problem into the modern era: what happens if we no longer marry men and women, but instead marry *anybody* to *anybody*?

Suppose we have a group of n people, where n is even. Each person ranks the other $n-1$ people in order of preference, and we want to find a stable matching. Note the difference in this problem formulation: we are no longer matching members of one group (e.g., men) to members of another group (e.g., women), but instead all members belong to the same single group (e.g., people).

Unfortunately, with this variant of the problem, a stable matching is not always guaranteed to exist! Show that this is the case by giving a counterexample of people and preference lists where no stable matching exists.

Hint. Matching two people is easy. What about four people?

[6 marks] 3. For each of the following blocks of pseudocode, give as tight a bound as possible on the time complexity of the pseudocode using Big-O notation. You do not need to give a formal proof, but you should give justifications for each of your answers. You may assume that n is a positive integer.

(a)
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z = 0
for i from 1 to n * n * n:
    for j from 1 to i * i:
        z = z + 1
```

(b)
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x = 0
for i from 1 to n + 23:
    x = x * 18
    for j from 355 to 2024:
        for k from 2i to 3i:
            x = x * 53
```

[6 marks] 4. Arrange the following functions in order from slowest growth rate to fastest growth rate. If function $f(n)$ is in position i of your list and function $g(n)$ is in position $i+1$ of your list, then this is equivalent to saying that $f(n) \in O(g(n))$. Give a brief justification for your ordering.

$$\begin{array}{ll} f_1 = n^3 & f_4 = 42^n \\ f_2 = \sqrt{6n} & f_5 = 2^{n^5} \\ f_3 = n - 111 & f_6 = \log(n) \cdot n^{2.5} \end{array}$$