St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Assignment 3
Due March 14, 2024 at 1:30pm

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[4 marks] 1. For each of the following recurrence relations, give an asymptotic tight bound in terms of n using the master theorem if possible, or otherwise indicate that the master theorem does not apply. Briefly justify your answers.

(a) $T(n) = 3T(n/3) + n/2$.

(b) $T(n) = 3T(n/2) + n$.

(c) $T(n) = 2T(n/4) + n^{0.51}$.

(d) $T(n) = 2^n T(n/2) + n^n$.

[6 marks] 2. Suppose you are given as input a sorted array A of n distinct integers, and you must determine whether or not there exists some index value i such that $A[i] = i$. Using the divide-and-conquer paradigm, design an algorithm that solves this problem in time $O(\log(n))$.

[7 marks] 3. In class, we saw how to select the kth smallest element from an array of n elements using the divide-and-conquer paradigm. We can use the same ideas to find other order statistics for a given array, such as the maximum or the minimum element.

Design an algorithm that uses the divide-and-conquer paradigm to compute the *maximum difference* between any two (not necessarily distinct) elements of an n-integer array. The maximum difference is the difference between the maximum and minimum elements of the array.

Your algorithm must run in $\Theta(n)$ time, and you should give a brief justification of its runtime. You may use the master theorem if you wish. Algorithms running in $\Theta(n \log(n))$ time will receive partial marks.

You may write your algorithm as pseudocode or you may implement your algorithm in the programming language of your choice. If you implement your algorithm, you must include a copy of your source code and an appropriate number of test cases with outputs demonstrating the correctness of your algorithm.

[8 marks] 4. The StFX recruitment department wants to try a new tactic for increasing application numbers: posting billboards along Highway 104 that advertise the university. The province has approved n sites along a K-kilometre stretch of highway where billboards may be posted, each labelled s_1 through s_n. Each of the sites s_i falls within the interval $[0, K]$.

If the university places a billboard at site s_i, it forecasts that it will receive $a_i > 0$ new applications. However, the province mandates that any two billboards along the highway must be placed more than 5 kilometres apart.

(a) Suppose $n = 4$, $K = 20$, $\{s_1, s_2, s_3, s_4\} = \{5, 8, 11, 15\}$, and $\{a_1, a_2, a_3, a_4\} = \{25, 30, 25, 5\}$. What is the subset of billboard sites that maximizes the number of new applications?

(b) Design an algorithm that takes as input integers $n > 0$ and $K > 0$, a set of sites $\{s_i\}$, and a set of new application forecasts $\{a_i\}$, and outputs the maximum number of new applications that arise from any valid subset of sites. Your algorithm must run in time polynomial in n.