St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Final Examination
April 21, 2023
9:00am–11:30am

Student Name: ________________________________

Email Address: ________________________________

Instructor: T. J. Smith (Section 20)

Format:

The exam is 150 minutes long. The exam consists of 6 questions worth a total of 70 marks. The exam booklet contains 9 pages, including the cover page and one blank page at the back of the exam booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your exam booklet contains 9 pages. Do not detach any pages from your exam booklet.
4. Do not use any unauthorized reference materials or devices during this exam.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: ________________________________

Question	Marks	Score
1	10	
2	10	
3	8	
4	16	
5	12	
6	14	
Total	70	

Multiple Choice

[10 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Which of the following functions $f(n)$ is **not** $O(g(n))$?

A. $f(n) = 2^n + n^7 + 2^7$, $g(n) = 2^{n^2}$

B. $f(n) = \log\left(\frac{n^2}{n-1}\right) + 5$, $g(n) = \log(n)$

C. $f(n) = n^2 + \frac{\sin(n)}{2}$, $g(n) = n$

D. $f(n) = \frac{17(n+\sqrt{n})}{3}$, $g(n) = n$

(b) Which of the following statements is **true**?

A. Greedy algorithms are guaranteed to provide an optimal solution for any problem.

B. Greedy algorithms make use of backtracking to find an optimal solution to a problem.

C. Greedy algorithms exhibit exponential time complexity in all cases.

D. Greedy algorithms make locally optimal choices that may not produce a globally optimal solution.

(c) Which of the following algorithms computes a minimum spanning tree by adding edges from least cost to greatest cost?

A. Prim's algorithm.

B. Kruskal's algorithm.

C. Borůvka's algorithm.

D. Hoare's algorithm.

(d) Which of the following statements best describes the idea behind a divide-and-conquer algorithm?

A. Solve a problem by dividing it into smaller subproblems and solving the subproblems iteratively.

B. Solve a problem by exploring all possible solutions in a systematic and exhaustive manner.

C. Solve a problem by making a locally optimal choice at each step in the hopes of finding the globally optimal solution.

D. Solve a problem by dividing it into smaller subproblems, solving the subproblems independently, and combining the solutions to construct a solution to the original problem.

(e) Consider the following recurrence relation:

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ 3T(n/2) + n^2 & \text{if } n > 1. \end{cases}$$

Using the master theorem, what can you conclude about the growth rate of $T(n)$?

A. $T(n) \in \Theta(n^{\log_2(3)})$.

B. $T(n) \in \Theta(n^2 \log(n))$.

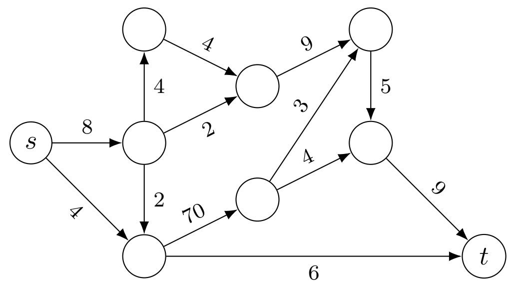
C. $T(n) \in \Theta(n^2)$.

D. None of the above.

(f) Which of the following algorithm design techniques uses memoization (i.e., caching the results of recursive function calls and returning cached results instead of recomputing values)?

A. Top-down dynamic programming.
B. Bottom-up dynamic programming.
C. Middle-out dynamic programming.
D. Upside-down dynamic programming.

(g) Consider the following flow network. What is the value of the maximum flow through this network?



A. 9
B. 11
C. 15
D. 70

(h) Which of the following statements is **true**?

A. In the reduction 3-SAT $\leq_{\text{P}}$ Directed-Hamiltonian-Cycle, if there are n variables in the original formula, the corresponding graph will contain 2^n Hamiltonian cycles.
B. Both the reductions 3-SAT $\leq_{\text{P}}$ Independent-Set and 3-SAT $\leq_{\text{P}}$ 3-Colour require us to connect three literals in a triangle in the corresponding graph structure.
C. The reductions 3-SAT $\leq_{\text{P}}$ Independent-Set and 3-SAT $\leq_{\text{P}}$ Vertex-Cover alone allow us to conclude that Independent-Set $\equiv_{\text{P}}$ Vertex-Cover.
D. In the reduction Directed-Hamiltonian-Cycle $\leq_{\text{P}}$ Hamiltonian-Cycle, we must construct two vertices for each vertex v in the original graph.

(i) Which of the following statements is **false**?

A. All problems in P can be reduced to 3-SAT.
B. All problems in NP can be reduced to 3-SAT.
C. All NP-complete problems can be reduced to 3-SAT.
D. All problems in EXP can be reduced to 3-SAT.

(j) Which of the following chains of inclusions is correct? (Note the difference between $\subset$ and $\subseteq$: the first is strict, while the second is not!)

A. $\mathsf{P} \subseteq \mathsf{NP} \subseteq \mathsf{EXP}$.
B. $\mathsf{P} \subset \mathsf{NP} \subseteq \mathsf{EXP}$.
C. $\mathsf{P} \subseteq \mathsf{NP} \subset \mathsf{EXP}$.
D. $\mathsf{P} \subset \mathsf{NP} \subset \mathsf{EXP}$.

Short Answer

[10 marks] 2. For each of the following questions, give a 1–2 sentence answer.

(a) What are the three strategies we learned for analyzing a greedy algorithm, and how do each of the strategies work?

(b) Why might we want to use Prim's algorithm to compute a minimum spanning tree over Kruskal's algorithm, or vice versa?

(c) What is the difference in the way a divide-and-conquer algorithm processes its input compared to a dynamic programming algorithm?

(d) Suppose we have a flow network for which we have found an *st*-cut with a capacity of 21, and we have proved that this is the smallest possible cut. What does this tell us about the value of the flow in the same network?

(e) A friend of yours has been learning about algorithms, and they told you that "any problem that has a polynomial-time algorithm is easy for us to solve". Is this a good working definition to use? Give one positive reason and one negative reason.

[8 marks] 3. Consider the recurrence relation $T(n) = 16T(n/4) + n^2$. For this recurrence relation,

- sketch the recurrence tree for $T(n)$;
- identify the values a, b, and c; and
- give an asymptotic bound on its growth rate using the master theorem.

[16 marks] 4. Recall the independent set problem: given an undirected graph $G = (V, E)$, a subset of vertices of G forms an independent set if no two vertices in the subset are adjacent to each other.

You may remember that the independent set problem is NP-complete. However, the problem becomes much easier to solve if we restrict ourselves to input graphs that are paths. A graph G is a path if it can be written as a sequence of vertices $\{v_1, v_2, \ldots, v_n\}$ with an edge between two vertices v_i and v_j if and only if $|i - j| = 1$.

Here, we consider the weighted path variant of the independent set problem: given a path graph G having a weight w_i associated with each vertex v_i, we want to find an independent set whose total weight is as large as possible.

(a) Your favourite professor claims that the following algorithm computes a solution to the weighted path independent set problem:

Algorithm: Weighted path independent set—heaviest-first

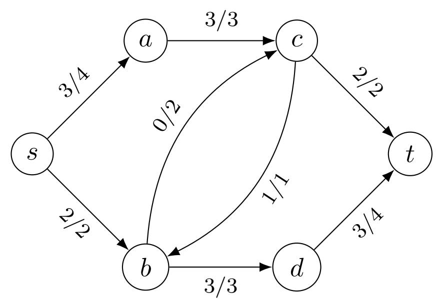
```
S ← ∅
while V is not empty do
    choose a vertex v_i having maximum weight
    S ← S ∪ {v_i}
    remove v_i and its neighbours from G
return S
```

However, your professor is wrong! Give an example of a path graph where this algorithm does *not* produce an independent set of maximum total weight.

(b) Using dynamic programming, design an algorithm that takes as input a path graph with n vertices and n weights, and returns an independent set of maximum total weight.

The runtime of your algorithm should be polynomial in the input size n and independent of the values of the weights, though you do not need to prove this.

[12 marks] 5. Consider the following flow network.



(a) Draw the residual network corresponding to the given flow network.

(b) What is the value of the flow shown in the network?

(c) Find a min cut in the flow network, and give the capacity of your min cut.

(d) Using your answer from part (c), can you conclude that the flow shown in the network is a max flow? Why or why not?

[14 marks] 6. Recall some of the NP-complete decision problems we defined in class:

A.	Independent set	D.	Knapsack
B.	Vertex cover	E.	3-colour
C.	Set cover	F.	Subset sum

Each of the problem statements below can be formulated as an instance of one of the above decision problems. For each problem statement below, match it to the most appropriate decision problem, and give a brief justification of your decision.

(a) The university's research office is deciding which projects to fund for the next academic year. Each project has a budget requirement expressed as some number of dollars. The government has provided a sum of money, but (i) the money is never adequate to fund all projects, and (ii) the university must use all the money, or the funding will be cut next year.

(b) You work as a security guard in an art gallery, and you are asked to figure out where to place some number of surveillance cameras within the gallery such that every corridor of the building is monitored. The gallery has a tight budget, though, so you must also minimize the number of surveillance cameras needed.

[2 marks] *Bonus.* What was your favourite part of this course, and why?

This blank page may be used for rough work.