St. Francis Xavier University
Department of Computer Science

CSCI 355: Algorithm Design and Analysis
Midterm Examination
February 9, 2023
1:15pm–2:05pm

Student Name: ______________________________

Email Address: ______________________________

Instructor: T. J. Smith (Section 20)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: ______________________________

Question	Marks	Score
1	5	
2	5	
3	7	
4	8	
Total	25	

Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Which of the following claims is **not** true about the Gale–Shapley algorithm?
   A. The algorithm produces a matching optimal for one of the two groups.
   B. The algorithm always produces a stable matching for all input instances.
   C. The algorithm can be used to solve the stable roommate problem.
   D. The algorithm has a runtime of $O(n^2)$ where n is the size of each group.

(b) What does it mean for a function $f(n)$ to be Big-Ω of another function $g(n)$?
   A. The function $g(n)$ is always larger than the function $f(n)$.
   B. For all large-enough values of n, $f(n)$ is bounded below by some constant times $g(n)$.
   C. We can replace $g(n)$ with $f(n)$ in any expression to produce an equivalent expression.
   D. For all large-enough values of n, some constant times $f(n)$ is bounded above by $g(n)$.

(c) Assuming that n is the size of the input, which of the following sequences correctly orders the given expressions from lowest to highest asymptotic growth rate?
   A. $2^{2023},\ n^{3/2},\ n\log(n),\ 3^n$
   B. $2^{2023},\ 3^n,\ n^{3/2},\ n\log(n)$
   C. $2^{2023},\ n\log(n),\ n^{3/2},\ 3^n$
   D. $n\log(n),\ n^{3/2},\ 2^{2023},\ 3^n$

(d) Which of the following is **not** a property of a bipartite graph?
   A. A bipartite graph may contain a cycle of even length.
   B. A bipartite graph is 2-colourable.
   C. A breadth-first search of a bipartite graph has no edge joining vertices in the same layer.
   D. A bipartite graph may contain a cycle of odd length.

(e) Which of the following algorithms solves the single-source shortest path problem?
   A. Dijkstra's algorithm.
   B. Kruskal's algorithm.
   C. Prim's algorithm.
   D. Smith's algorithm.

Short Answer

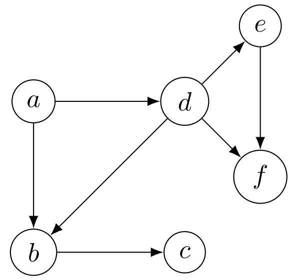
[5 marks] 2. Consider the following pseudocode. State the runtime of this algorithm in terms of Big-Θ notation, assuming n is the input size. Also give a brief (2–3 sentence) justification of your answer.

You can assume that assignments and arithmetic operations take $\Theta(1)$ time. Note that you should give the tightest bound possible as your answer.

Algorithm: Mystery algorithm

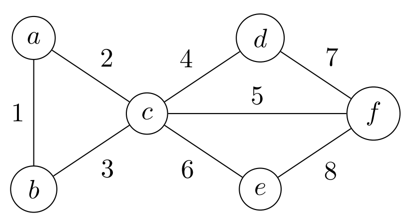
```
while i ≤ n do
    j ← 1
    while j ≤ n do
        j ← j · 2
        print i · j
    i ← i + 1
```

[7 marks] 3. (a) Consider the following graph G. Give a topological ordering of G.



(b) Using any appropriate algorithm, find a minimum spanning tree for the following graph.

Specify which algorithm you are using. You do not need to draw every step performed by the algorithm; you only need to give the sequence of vertices or edges added by the algorithm. For clarity, draw your final minimum spanning tree on the graph as well.



[8 marks] 4. The university's IT Services group needs to purchase licences for n different pieces of software. Due to budgeting rules, however, they can only purchase at most one licence per month.

Each licence currently sells for \$100, but the cost is marked up each month. Specifically, the cost of licence j increases by a factor of $r_j > 1$ each month. Therefore, if licence j is purchased t months from now, it will cost $\$100 \cdot {r_j}^t$. You can assume that $r_i \neq r_j$ for all $i \neq j$.

IT Services claims that their algorithm, which sorts the r_j values in decreasing order to determine the order in which to purchase each licence j, minimizes the total cost.

Prove that this algorithm is indeed optimal using an exchange argument.

This blank page may be used for rough work.