St. Francis Xavier University
Department of Computer Science

CSCI 544: Computational Logic
Assignment 1
Due February 9, 2024 at 11:30am

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. (a) Translate each of the following English sentences into the syntax of propositional logic using the indicated letters. For each letter, indicate explicitly the statement it stands for.

i. Earning a final mark of 70% in the computational logic course is a necessary condition for a student to obtain their master's degree. (ℓ, p)

ii. If computational logic is a difficult subject, then Bobby and Connie will pass only if they study. (d, b, c, s)

iii. If Q is a quadrilateral, then Q is a parallelogram if and only if its opposite sides are both equal and parallel. (q, p, e, ℓ)

(b) Translate the following argument from Plato's *Crito* into the syntax of propositional logic, and then prove that the argument is valid.

Premise 1. If Socrates did not approve of the laws of Athens, then he would either have left Athens or else have tried to change the laws.

Premise 2. If Socrates did not leave Athens and did not try to change the laws, then he agreed to obey the laws.

Premise 3. Socrates did not leave Athens.

Conclusion. If Socrates did not try to change the laws, then he approved of the laws and agreed to obey them.

Hint. You can prove validity by contradiction: assume that there exists an interpretation where all premises are true but the conclusion is false.

[5 marks] 2. Consider the formula

$$\neg(s \Rightarrow (\neg(p \Rightarrow (q \vee \neg s)))).$$

Draw the parse tree for this formula, and list all subformulas that appear in the formula.

[6 marks] 3. (a) Let $\oplus$ denote the logical connective of *exclusive disjunction* (i.e., XOR). This connective is defined as $r \oplus s \equiv (r \vee s) \wedge \neg(r \wedge s)$. Prove that, for all propositions r and s, $((r \oplus s) \oplus s) \equiv r$.

(b) Let $\uparrow$ denote the logical connective of *alternative denial* (i.e., NAND). This connective is defined as $p \uparrow q \equiv \neg(p \wedge q)$. Prove that, for all propositions p and q, $(p \uparrow q) \uparrow (p \uparrow q) \equiv p \wedge q$.

[6 marks] 4. Using the method of semantic tableaux, determine whether each of the following formulas is valid. If a formula is not valid, give an example of an interpretation demonstrating this. (Remember that a formula A is valid if and only if $\neg A$ is unsatisfiable.)

(a) $A = (p \vee q) \Rightarrow (p \wedge r)$.

(b) $B = (p \Rightarrow q) \vee (q \Rightarrow p)$.

[10 marks] 5. Prove the validity of each of the following sequents using natural deduction.

(a) $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$.

(b) $p \Rightarrow q, r \Rightarrow s \vdash p \wedge r \Rightarrow q \wedge s$.

[5 marks] 6. Using natural deduction, prove the following:

$$(p \Rightarrow q) \Rightarrow ((\neg p \Rightarrow q) \Rightarrow q).$$

Hint. You may need to use the *Law of Excluded Middle* in your proof: $p \vee \neg p$.