# St. Francis Xavier University
# Department of Computer Science

## CSCI 544: Computational Logic
## Assignment 2
## Due March 15, 2024 at 11:30am

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**Assignment Regulations.**

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

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[8 marks] 1. Consider the formula

$$A = (\forall x\ \exists z\, (\neg P(x) \vee Q(y, f(z)))) \Rightarrow (\exists x\ P(y) \wedge Q(f(x), z))\,.$$

(a) Draw the tree representation of $A$.

(b) Identify all bound variables and free variables in $A$. Does there exist any variable in $A$ that has both bound and free occurrences?

(c) Give the resultant formula after performing the substitution $A[y/f(x)]$. Is $f(x)$ free for $y$ in $A$?

[6 marks] 2. Consider the following three mathematical properties, expressed as predicate logic formulas:

| | |
|---|---|
| Reflexivity: | $\forall x\ P(x, x)$ |
| Symmetry: | $\forall x\ \forall y\ (P(x, y) \Rightarrow P(y, x))$ |
| Transitivity: | $\forall x\ \forall y\ \forall z\ ((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z))$ |

Give an interpretation (i.e., a domain, a binary relation $P$, and any necessary constants) where the binary relation is:

(a) Reflexive and symmetric, but not transitive.

(b) Reflexive and transitive, but not symmetric.

(c) Symmetric and transitive, but not reflexive.

[8 marks] 3. Using the method of semantic tableaux, determine whether the following formula is valid:

$$(\forall x\ (A(x) \Rightarrow B(x))) \Rightarrow (\exists x\ A(x) \Rightarrow \exists x\ B(x)).$$

For ease of writing, you can use the list format to prove validity instead of the tree format.

[8 marks] 4. Prove the validity of each of the following sequents using natural deduction.

(a) $\forall x\ (P(x) \wedge Q(x)) \vdash \forall x\ P(x) \wedge \forall x\ Q(x)$.

(b) $\forall x\ \neg P(x) \vdash \neg \exists x\ P(x)$.