MATHEMATICAL GAMES

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Source: *Scientific American*, Vol. 202, No. 2 (February 1960), pp. 150-156
Published by: Scientific American, a division of Nature America, Inc.
Stable URL: https://www.jstor.org/stable/10.2307/24941266

MATHEMATICAL GAMES

A fifth collection of "brain-teasers"

by Martin Gardner

Every eight months or so this department presents an assortment of short problems drawn from various mathematical fields. This is the fifth such collection. The answers to the problems will be given here next month. I welcome letters from readers who find fault with an answer, solve a problem more elegantly, or generalize a problem in some interesting way. In the past I have tried to avoid puzzles that play verbal pranks on the reader, so I think it only fair to say that several of this month's "brain-teasers" are touched with whimsy. They must be read with care; otherwise you may find the road to a solution blocked by an unwarranted assumption.

1.

Mel Stover of Winnipeg was the first to send this amusing problem—amusing because of the ease with which even the best of geometers may fail to approach it properly. Given a triangle with one obtuse angle, is it possible to cut the triangle into smaller triangles, all of them acute? (An acute triangle is a triangle with three acute angles. A right angle is of course neither acute nor obtuse.) If this cannot be done, give a proof of impossibility. If it can be done, what is the smallest number of acute triangles into which any obtuse triangle can be dissected?

The illustration at right shows a typical attempt that leads nowhere. The triangle has been divided into three acute triangles, but the fourth is obtuse, so nothing has been gained by the preceding cuts.

This delightful problem led me to ask myself: "What is the smallest number of acute triangles into which a square can be dissected?" For days I was convinced that nine was the answer; then suddenly I saw how to reduce it to eight. I wonder how many readers can discover an eight-triangle solution, or perhaps an even better one. I am unable to prove that eight is the minimum, though I strongly suspect that it is.

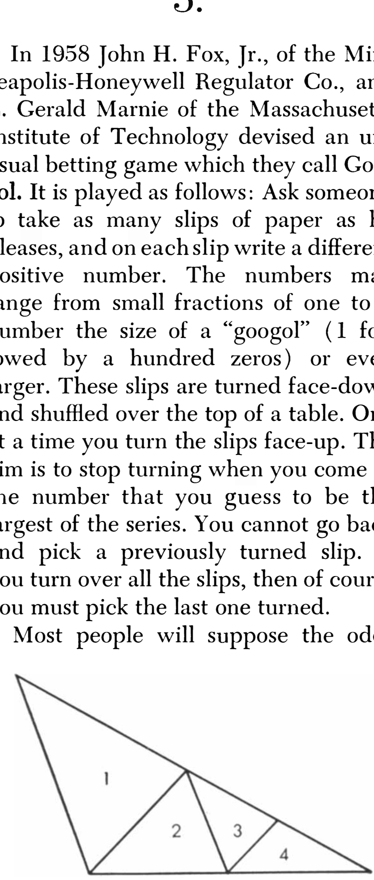
2.

In H. G. Wells's novel *The First Men in the Moon* our natural satellite is found to be inhabited by intelligent insect creatures who live in caverns below the surface. These creatures, let us assume, have a unit of distance that we shall call a "lunar." It was adopted because the moon's surface area, if expressed in square lunars, exactly equals the moon's volume in cubic lunars. The moon's diameter is 2,160 miles. How many miles long is a lunar?

3.

In 1958 John H. Fox, Jr., of the Minneapolis-Honeywell Regulator Co., and L. Gerald Marnie of the Massachusetts Institute of Technology devised an unusual betting game which they call Googol. It is played as follows: Ask someone to take as many slips of paper as he pleases, and on each slip write a different positive number. The numbers may range from small fractions of one to a number the size of a "googol" (1 followed by a hundred zeros) or even larger. These slips are turned face-down and shuffled over the top of a table. One at a time you turn the slips face-up. The aim is to stop turning when you come to the number that you guess to be the largest of the series. You cannot go back and pick a previously turned slip. If you turn over all the slips, then of course you must pick the last one turned.

Most people will suppose the odds



Can this triangle be cut into acute ones?

This common game is a sequential machine. Its input is the way you move the numbered tiles. Output is the arrangement of numbers you produce.

States in a Sequential Machine

A "sequential machine" is any device producing prescribed sequences of outputs in response to given sequences of inputs. The theoretical problem of designing a machine, satisfying certain specifications with the fewest possible number of states, is now under study by IBM scientists.

The operation of a sequential machine is not necessarily completely specified. Some states may have no specified transitions for certain inputs, and some states may have no assigned outputs. For this general case, a technique has been developed for reducing a given machine to an equivalent machine with a minimum number of states. The procedure is to construct a state diagram of the machine which describes input and output sequences. Then through the use of a transition-matrix representation, a minimum-state diagram is obtained, which is equivalent to the original machine in the sense that it will produce the same sequences of outputs for the given sequence of inputs.

Earlier reduction procedures have been applicable only to state-diagrams having known transitions for each input at each state. The extension of the procedure is important since many practical sequential machines (such as computers) require a specified operation for only a certain set of sequences of inputs.

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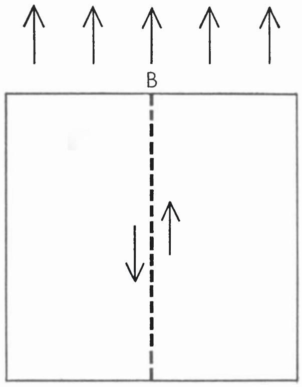


How far does the dog trot?

against your finding the highest number to be at least five to one. Actually if you adopt the best strategy, your chances are a little better than one in three. Two questions arise. First, what is the best strategy? (Note that this is not the same as asking for a strategy that will maximize the *value* of the selected number.) Second, if you follow this strategy, how can you calculate your chances of winning?

When there are only two slips, your chance of winning is obviously 1/2, regardless of which slip you pick. As the slips increase in number, the probability of winning (assuming that you use the best strategy) decreases, but the curve flattens quickly, and there is very little change beyond 10 slips. The probability never drops below 1/3. Many players will suppose that they can make the task more difficult by choosing very large numbers, but a little reflection will show that the sizes of the numbers are irrelevant. It is only necessary that the slips bear numbers that can be arranged in increasing order.

The game has many interesting applications. For example, a bachelor girl decides to marry before the end of the year. She estimates that she will meet 10 men who can be persuaded to propose, but once she has rejected a proposal, the man will not try again. What strategy should she follow to maximize her chances of accepting the top man of the 10, and what is the probability that she will succeed?

The strategy consists of rejecting a certain number of slips of paper (or proposals), then picking the next number that exceeds the highest number among the rejected slips. What is needed is a formula for determining how many slips to reject, depending on the total number of slips. This is not an easy problem, but readers who find it too difficult may enjoy playing the game with, say, 10 slips to see how close they can come by trial and error to guessing the number of slips that should be rejected to maximize the chances of success.

4.

A square formation of Army cadets, 50 feet on the side, is marching forward at a constant pace [*see illustration on this page*]. The company mascot, a small terrier, starts at the center of the rear rank [*position A in the illustration*], trots forward in a straight line to the center of the front rank [*position B*], then trots back again in a straight line to the center of the rear. At the instant he returns to position A, the cadets have advanced exactly 50 feet. Assuming that the dog trots at a constant speed and loses no time in turning, how many feet does he travel?

If you solve this problem, which calls for no more than a knowledge of elementary algebra, you may wish to tackle a much more difficult version proposed by the famous puzzlist Sam Loyd. Instead of moving forward and back through the marching cadets, the mascot trots with constant speed around the *outside* of the square, keeping as close as possible to the square at all times. (For the problem we assume that he trots along the perimeter of the square.) As before, the formation has marched 50 feet by the time the dog returns to point A. How long is the dog's path? I know of no way to solve Loyd's version without getting tangled in fifth-degree equations. The method of solution is so lengthy to describe that next month I shall give only the answer. I shall return to the problem, however, if any reader discovers a simple method of solving it.

5.

Stephen Barr of Woodstock, N.Y., writes that his dressing gown has a long cloth belt, the ends of which are cut at 45-degree angles as shown in the illustration on the next page. When he packs the belt for a trip, he likes to roll it up as neatly as possible, beginning at one end, but the slanting ends disturb his sense of symmetry. On the other hand, if he folds over an end to make it square off, then the uneven thicknesses of cloth put lumps in the roll. He experimented with more complicated folds, but try as he would, he could not achieve a rectangle of uniform thickness. For example, the fold shown in the illustration produces a rectangle with three thicknesses in section A and two in section B.

"Nothing is perfect," says one of the

Philosophers in James Stephens's *The Crock of Gold*. "There are lumps in it." Nonetheless Barr finally managed to fold his belt so that each end was straight across and part of a rectangle of uniform thickness throughout. The belt could then be folded into a neat roll, free of lumps. How did Barr fold his belt? A long strip of paper, properly cut at the ends, can be used for working on the problem.

6.

Professor Merle White of the mathematics department, Professor Leslie Black of philosophy, and Jean Brown, a young stenographer who worked in the university's office of admissions, were lunching together.

"Isn't it remarkable," observed the lady, "that our last names are Black, Brown and White and that one of us has black hair, one brown hair and one white."

"It is indeed," replied the person with black hair, "and have you noticed that not one of us has hair that matches his or her name?"

"By golly, you're right!," exclaimed Professor White.

If the lady's hair isn't brown, what color is it?

7.

An airplane, traveling with a constant engine speed, makes a large and perfect circle parallel to the ground. There is no wind. Will it complete the same circle in greater, less or the same time if there is a wind of constant speed and direction, assuming that the plane travels with the same constant engine speed that it had before?

8.

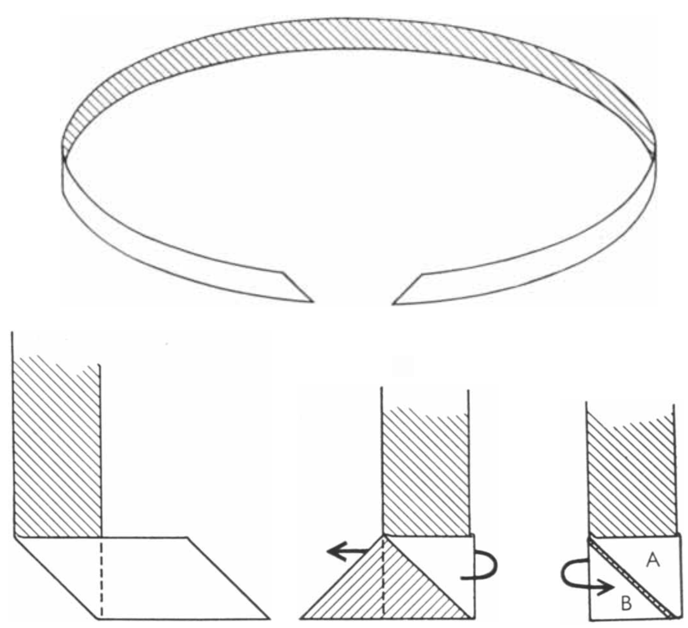
The owner of a pet shop bought a certain number of hamsters and half that many pairs of parakeets. He paid $2 each for the hamsters and $1 for each parakeet. On every pet he placed a retail price that was an advance of 10 per cent over what he paid for it.

After all but seven of the creatures had been sold, the owner found that he had taken in for them an amount of money exactly equal to what he had originally paid for all of them. His profit, therefore, was represented by the combined retail value of the seven remaining animals. What was this value?

The two numerology problems posed last month by Dr. Matrix are answered as follows:

(1) The letters OTTFFSSENT are the initials of the names of the cardinal numbers from one to 10.

(2)	FORTY	29786
	+ TEN	850
	+ TEN	850
	SIXTY	31486



Barr's belt (top) *and an unsatisfactory way to fold it* (bottom)

FROM LAGRANGIAN TO LIFT-OFF

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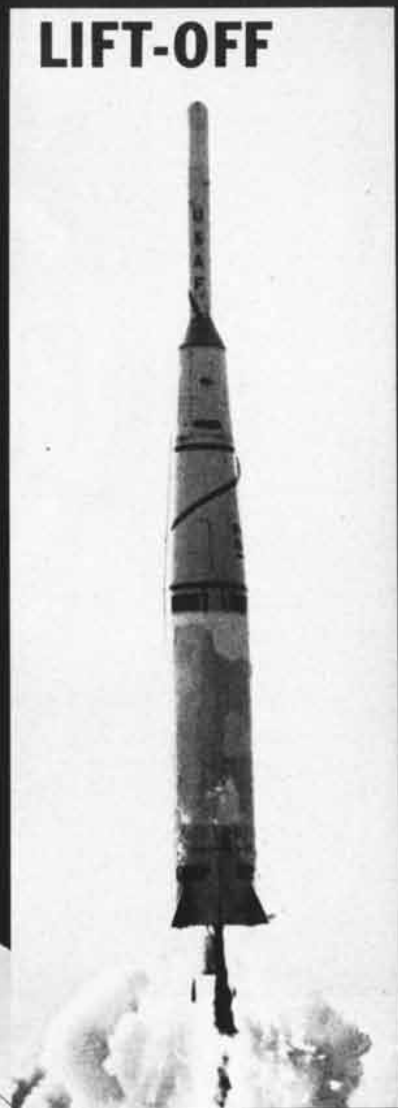

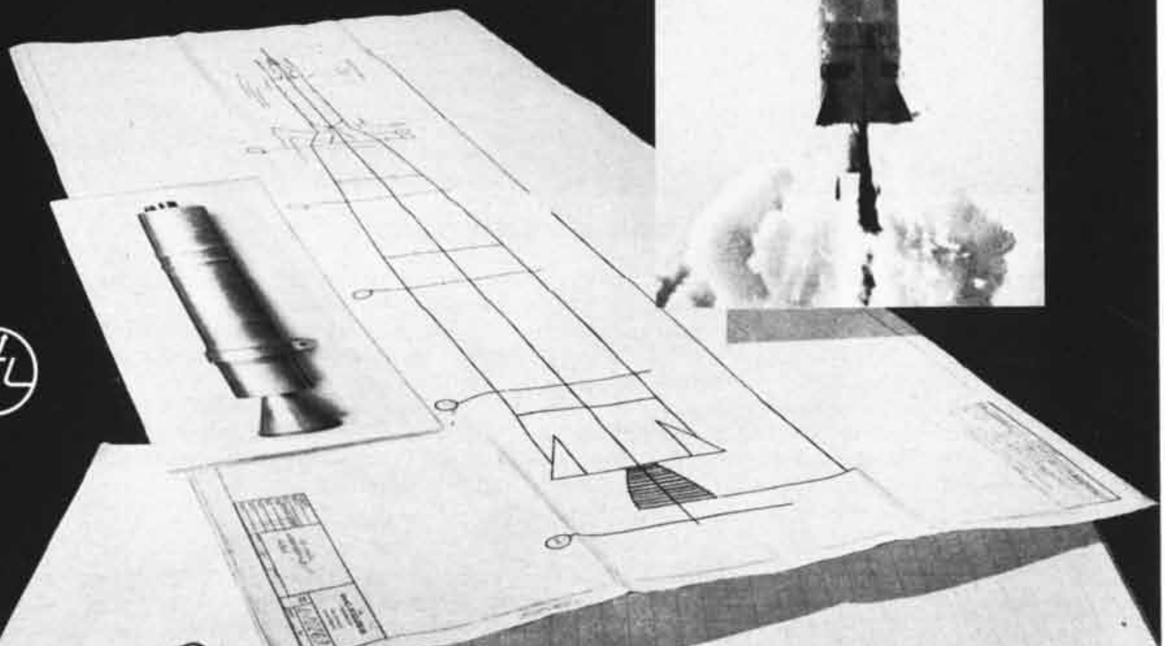

$$\mathcal{L} = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right) + \frac{GMm}{r}$$