St. Francis Xavier University
Department of Computer Science

CSCI 435: Algorithms and Complexity
Midterm Examination
February 15, 2023
10:15am–11:05am

Student Name: ______________________________

Email Address: ______________________________

Instructor: T. J. Smith (Section 20)

Format:

The midterm is fifty minutes long. The midterm consists of 4 questions worth a total of 25 marks. The midterm booklet contains 6 pages, including the cover page and one blank page at the back of the midterm booklet for rough work.

Reference Materials:

None.

Instructions:

1. Write your name and email address in the spaces above.
2. Answer each question either in the space provided or on a blank page. If you use a blank page to write your answer, indicate this clearly in the space provided for the question. Show all of your work.
3. Ensure that your midterm booklet contains 6 pages. Do not detach any pages from your midterm booklet.
4. Do not use any unauthorized reference materials or devices during this midterm.
5. Sign in the space below. Your signature indicates that you understand and agree to these instructions and the university's examination policies.

Signature: ______________________________

Question	Marks	Score
1	5	
2	6	
3	8	
4	6	
Total	25	

Multiple Choice

[5 marks] 1. For each of the following questions, select exactly one answer by circling the associated letter. Incorrect answers will not be penalized. Answers with more than one letter circled will be marked as incorrect.

(a) Consider the following recurrence relation:

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ 2T(n/2) + n & \text{if } n > 1. \end{cases}$$

Using the master theorem, what can you conclude about the growth rate of $T(n)$?

A. $T(n) \in \Theta(n)$.
B. $T(n) \in \Theta(n \log(n))$.
C. $T(n) \in \Theta(2^n)$.
D. $T(n) \in \Theta(2T(n/2))$.

(b) What does it mean for an online algorithm to be α-competitive?

A. $\mathrm{ALG}(\sigma) \leq \alpha \cdot \mathrm{OPT}(\sigma)$ for all sequences of requests σ.
B. $\alpha \cdot \mathrm{ALG}(\sigma) \geq \mathrm{OPT}(\sigma)$ for all sequences of requests σ.
C. $\mathrm{ALG}(\sigma)$ outperforms $\mathrm{OPT}(\sigma)$ for all sequences of requests σ with $|\sigma| \leq \alpha$.
D. The online algorithm performs at most α comparisons during its computation.

(c) Which of the following algorithms for the paging problem is **not** α-competitive for any value of α?

A. Random
B. First In First Out
C. Least Recently Used
D. Last In First Out

(d) Suppose we design an algorithm that attempts to return the lucky number 7 in the following way: the algorithm iterates five times and, on each iteration, it generates a random integer between 0 and 10. If 7 is generated on any iteration, the algorithm returns it; otherwise, after five iterations, the algorithm returns "unlucky". What kind of randomized algorithm is this?

A. Las Vegas
B. Monte Carlo with two-sided error
C. Monte Carlo with one-sided error
D. Atlantic City

(e) Which of the following is the correct ordering of the given complexity classes?

A. $\mathsf{P} \subseteq \mathsf{BPP} \subseteq \mathsf{ZPP} \subseteq \mathsf{PP}$
B. $\mathsf{P} \subseteq \mathsf{ZPP} \subseteq \mathsf{PP} \subseteq \mathsf{BPP}$
C. $\mathsf{P} \subseteq \mathsf{PP} \subseteq \mathsf{ZPP} \subseteq \mathsf{BPP}$
D. $\mathsf{P} \subseteq \mathsf{ZPP} \subseteq \mathsf{BPP} \subseteq \mathsf{PP}$

Short Answer

[6 marks] 2. Suppose we have n microchips that we received from a factory of questionable quality. Some of the chips are working, while others are faulty. These chips can communicate with each other and report whether other chips are functioning correctly: if we take a pair of chips, a working chip always reports accurately whether the other chip is working or faulty, but a faulty chip gives an untrustworthy report. Thus, we have four testing outcomes:

Chip A's Report	**Chip B's Report**	**Conclusion**
B is working	A is working	Both are working or both are faulty
B is working	A is faulty	At least one is faulty
B is faulty	A is working	At least one is faulty
B is faulty	A is faulty	At least one is faulty

Assuming that more than $n/2$ of the chips are working, describe a procedure to find a single guaranteed-working chip from our shipment of n chips.

[8 marks] 3. (a) State the MARKING algorithm for the paging problem. You may choose to write the pseudocode for the algorithm, describe the algorithm as a series of steps, or describe the algorithm in plain English.

(b) Consider an instance of the paging problem where the size of the cache is $k = 3$ and the number of pages to be accessed is $n = 8$. Give the sequence of cache states produced by the MARKING algorithm by the following sequence of page access requests:

$$\sigma = \{p_1, p_2, p_3, p_5, p_3, p_4, p_3, p_1\}.$$

Note. Show all your work, including all intermediate steps and marking/unmarking steps.

[6 marks] 4. Suppose we are given a graph $G = (V, E)$ and we colour each vertex of G with one of three colours (say: red, green, or blue). In a 3-colouring of G, we say that an edge (u, v) in G is *satisfied* if the colours assigned to vertices u and v are different.

Design a randomized polynomial-time algorithm that takes as input a graph G and produces a 3-colouring where the expected number of satisfied edges is $2/3$ of the total edges in G, and explain how your algorithm achieves this expected value.

This blank page may be used for rough work.