St. Francis Xavier University
Department of Computer Science

CSCI 435: Algorithms and Complexity
Space Complexity — Discussion Questions
Winter 2025

[Mic92] Pascal Michel. A Survey of Space Complexity. *Theoretical Computer Science*, 101:99–132, 1992.

Pascal Michel is a retired researcher, previously of the Equipe de Logique mathématique at L'Institut de mathématiques de Jussieu – Paris Rive Gauche. His work spans theoretical computer science, computational complexity, and the complexity of logical theories, with a particular emphasis on the famous "busy beaver" function.

1. What does it mean for a Turing machine to "work in space $S(n)$"? Does the space usage of a Turing machine depend on the number of tapes it has? Why or why not?
2. What makes a function space constructible? Give an informal argument of why the function n^2 is space constructible.

   (*Hint.* Suppose we have n symbols written to the input tape. How can we use this to write n^2 symbols to another tape?)
3. Recall some basic complexity classes like P, NP, PSPACE, and EXP. How can we define these classes in terms of DTIME, NTIME, DSPACE, and NSPACE?
4. Recall the language classes REG and CFL. How can we define REG in terms of DSPACE? (In other words, how much space do we require to recognize a regular language?) How can we define CFL in terms of DTIME? (In other words, how much time does it take to recognize/parse a context-free language?)

[ADGV15] Greg Aloupis, Erik D. Demaine, Alan Guo, and Giovanni Viglietta. Classic Nintendo Games Are (Computationally) Hard. *Theoretical Computer Science*, 586:135–160, 2015.

Erik Demaine is a professor of computer science at MIT. He was born in Halifax, and completed his bachelor's degree at Dalhousie at the age of 14! Much of his work focuses on two areas: computational geometry, and (as indicated by the present paper) the computational complexity of games. Greg Aloupis, Alan Guo, and Giovanni Viglietta are just a few of his many collaborators on research in this area.

1. What is the definition of the decision problem TQBF? How does it relate to other decision problems we have seen, such as SAT?
2. What is a "gadget" in the context of this paper? How can we use gadgets to prove hardness results?
3. Choose your favourite video game from this paper, and give a high-level description of the proof of that game's hardness. (You do not have to go into a great amount of detail, but you should make the key ideas of the proof clear.)