St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 1
Due February 6, 2025 at 1:30pm

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Prove that a deterministic one-tape Turing machine $\mathcal{M}_1$ is capable of recognizing every language that a deterministic k-tape Turing machine $\mathcal{M}_k$ can recognize.

You must give a complete description of exactly how $\mathcal{M}_1$ simulates the computation of $\mathcal{M}_k$, including how the contents of all k tapes are stored on the single tape of $\mathcal{M}_1$.

[10 marks] 2. Construct a single-tape deterministic Turing machine $\mathcal{M}$ that performs the following "right shift" operation: given a word $w \in \{\mathtt{a}, \mathtt{b}\}^*$, $\mathcal{M}$ shifts the entire input word rightward by one cell on its input tape. After performing this shift, $\mathcal{M}$ halts and accepts.

For example, if $\mathcal{M}$ is given the input word `abba`, its tape at the beginning of the computation will look like

a	b	b	a	␣	␣	␣	⋯

and, after $\mathcal{M}$ halts and accepts, its tape will look like

␣	a	b	b	a	␣	␣	⋯

You should give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$. You may optionally draw the Turing machine to illustrate your construction.

[8 marks] 3. Prove that, if $f(n) \geq n$ and $g(n) \geq n$ are both time constructible functions, then the functions $f(n)+g(n)$ and $f(n) \cdot g(n)$ are also time constructible.

[8 marks] 4. Show that the function 2^n is time constructible. To do this, you must explain how to construct a Turing machine that takes as input the word $\mathtt{1}^n$ and writes the word $\mathtt{1}^{2^n}$ to its output tape in time $O(2^n)$.

[6 marks] 5. For each of the following questions, use the appropriate hierarchy theorem to prove the given statement.

(a) Prove that $\mathsf{DTIME}(n^3) \subset \mathsf{DTIME}(n^3 \cdot \sqrt{n})$.

(b) Prove that $\mathsf{DSPACE}(2^n) \subset \mathsf{DSPACE}(2^n \cdot \sqrt{n})$.