St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 1
Due February 5, 2026 at 1:30pm

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Construct a single-tape deterministic Turing machine $\mathcal{M}$ that recognizes the following language L over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

$$L = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 1\}.$$

You should give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$. Each of your transitions should be written in the form $\delta(q, a) = (q', a', D)$, where $q, q' \in Q$, $a, a' \in \Gamma$, and $D \in \{L, R\}$.

You may optionally draw the Turing machine to illustrate your construction.

[10 marks] 2. A deterministic k-tape Turing machine without leftward moves has its transition function restricted in the following way: in any computation step, the input head and the work tape heads can either make a rightward move or remain stationary (i.e., make no move). Each tape head can make its move independently of the others. Formally, the transition function is defined to be

$$\delta\colon Q \times \Gamma^k \to Q \times \Gamma^{k-1} \times \{R, N\}^k.$$

(a) Prove that deterministic k-tape Turing machines without leftward moves *cannot* simulate the computation of an ordinary deterministic k-tape Turing machine.

(b) What class of languages does the usual Turing machine model recognize, and what class of languages does a Turing machine without leftward moves recognize? (You may recall this from a past course, or otherwise you may need to do some research.)

[8 marks] 3. Prove that, if $f(n) \geq n$ and $g(n) \geq n$ are both time constructible functions, then the functions $f(n)+g(n)$ and $f(n) \cdot g(n)$ are also time constructible.

[8 marks] 4. Show that the function 2^n is time constructible. To do this, you must explain how to construct a Turing machine that takes as input the word $\mathtt{1}^n$ and writes the word $\mathtt{1}^{2^n}$ to its output tape in time $O(2^n)$.

[6 marks] 5. For each of the following questions, use the appropriate result or hierarchy theorem to prove the given statement.

(a) Prove that $\mathsf{DSPACE}(n^5) \supseteq \mathsf{NTIME}(n^2 \log(n))$.

(b) Prove that $\mathsf{DTIME}(n^3) \subset \mathsf{DTIME}(n^3 \cdot \sqrt{n})$.

(c) Let g be a function defined as follows: $g(n) = n$ when n is even and $g(n) = n^3$ when n is odd. Prove that $\mathsf{DTIME}(g(n))$ and $\mathsf{DTIME}(n^2)$ are incomparable.