St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 2
Due March 12, 2026 at 1:30pm

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[6 marks] 1. Let $\mathsf{LINSPACE} = \mathsf{DSPACE}(n)$ denote the class of decision problems solvable in deterministic linear space (i.e., $O(n)$ space). Prove that $\mathsf{P} \neq \mathsf{LINSPACE}$.

Hint. Use proof-by-contradiction and the space hierarchy theorem.

[8 marks] 2. Prove that if $\mathsf{NSPACE}(n) \subseteq \mathsf{NP}$, then $\mathsf{NP} = \mathsf{PSPACE}$.

Hint 1. What do we know about the relationship between NP and PSPACE?

Hint 2. Use Savitch's theorem.

[10 marks] 3. (a) Prove that every PSPACE-hard decision problem is also NP-hard.

(b) Suppose that every NP-hard decision problem were also PSPACE-hard. What would this imply? Explain your reasoning.

[10 marks] 4. Suppose you have a Las Vegas randomized algorithm solving a given problem in expected polynomial time. Show that you can always convert this to a Monte Carlo randomized algorithm solving the same problem with one-sided error in polynomial time.

Hint 1. Remember that your Monte Carlo algorithm must produce a correct answer with probability $\geq 1/2$. This probability can be modelled by the expression $\mathbb{P}[T_{\mathrm{LV}}(n) < T_{\mathrm{MC}}(n)]$, where $T_{\mathrm{LV}}(n)$ is the runtime of the Las Vegas algorithm and $T_{\mathrm{MC}}(n)$ is the runtime of the Monte Carlo algorithm.

Hint 2. Building on the previous hint, you may find *Markov's inequality* useful: given a nonnegative random variable X and a value $a > 0$, we have that $\mathbb{P}[X \geq a] \leq \mathbb{E}[X]/a$.

[6 marks] 5. Recall the definition of the class coRP:

$$\mathsf{coRP} = \{\Sigma^* \setminus L \mid L \subseteq \Sigma^*, L \in \mathsf{RP}\}.$$

Prove that if $\mathsf{coRP} \subseteq \mathsf{RP}$, then $\mathsf{RP} = \mathsf{coRP}$.